## Kangaroo of Mathematics 2009 Level Junior (Grades 9 and 10) Austria - 23.3.2009



- 3 Points Questions -1) Which of the following number is divisible by 3? **B**) 2 + 0 + 0 + 9**D**)  $2^9$ **C**) 200 ó 9 **E**)  $(2+0) \cdot (0+9)$ A) 2009 2) What is the minimum number of points which have to be removed from the adjacent diagram so that in the remaining picture no three points lie in one line? **C**) 3 **D**) 4 **A**) 1 **B**) 2 **E**) 7 3) 2009 people are taking part in a public fun run. The number of people Hans beat is three times as big as the number of people that have finished before him. In which place did Hans finish the race? **A)** 503. **B**) 501. **C**) 500. **D**) 1503. **E)** 1507. 4) Harry does his paper round in Long Street. He has to deliver one newspaper to each house with an odd number. The first house with an odd number is 15 and the final house has the number 53. How many houses does Harry have to visit? **A)** 19 **B**) 20 **C)** 27 **D**) 38 **E**) 53 5) The product of four different natural numbers is 100. What is the sum of the four numbers? **A)** 10 **B)** 12 **C)** 15 **D**) 18 **E)** 20 6) What is the value of  $\frac{1}{2} \cdot \frac{2}{3} \cdot \frac{3}{4} \cdot \frac{4}{5} \cdot \frac{5}{6} \cdot \frac{6}{7} \cdot \frac{7}{8} \cdot \frac{8}{9} \cdot \frac{9}{10} \cdot 1000$ ? **A)** 250 **B**) 200 **C)** 100 **D**) 50 **E**) a different number 7) A long number is build by writing the number 2009, 2009 times in a line. How big is the sum off all odd number within this number that are immediately to the left of an even number? **A**) 2 **B**) 9 **C)** 4018 **D**) 18072 **E)** 18081 8) The diagram on the right shows a solid made up of 6 triangles. Each vertex is assigned a number, two of which are indicated. The total of the three numbers on each triangle is the same. What is the total of all five numbers? 5 **A**) 9 **B**) 12 **C)** 17 **D**) 18 **E)** 24

9) For how many positive whole numbers a do a<sup>2</sup> and a<sup>3</sup> have the same amount of digits?

**A**) 0 **B**) 3 **C**) 4 **D**) 9 **E**) infinite many

10) The area of the shown triangle equals  $80m^2$ . Each circle has a radius of 2m and itøs centre is in one of the vertices of the triangles. What is the area of the grey shaded region (in  $m^2$ )?

**A)** 76 **B)**  $80 - 2\pi$  **C)**  $40 - 4\pi$  **D)**  $80 - \pi$  **E)**  $78\pi$ 



11) Leonhard made up a sequence where, starting with the third term, each term is the sum of the previous two numbers. The fourth number is 6 and the sixth number is 15. What is the seventh number in the sequence?

**A**) 9 **B**) 16 **C**) 21 **D**) 22 **E**) 24

12) In the triangle illustrated one internal angle measures 68°. The three angle bisectors of the triangle are shown. What is the size of the angle indicated with a question mark?



**A)** 120° **B)** 124° **C)** 128° **D)** 132° **E)** 136°

13) Maria can achieve 0, 1, 2, 3, 4 or 5 points in a test. After 4 tests she has a mean of exactly 4. One of the following sentences can therefore not be true. Which one is it?

A) Maria achieved 4 in each test. B) Maria achieved 3 exactly twice.

C) Maria achieved 4 exactly twice. D) Maria achieved 1 exactly once.

E) Maria achieved 3 exactly three times.

14) The õBorromaic Ringsö have an extraordinary property. Although no two are interlocked, they are strongly connected within each other. If one ring is cut through, the other two fall apart. Which of the following diagrams shows the picture of õBorromaic Ringsö?



15) On the island of the nobles and liars 25 people are stanging in a queue. The first person in the line claims that everybody behind him is a liar. Each of the other people claims that the person in front of him is a liar. How many liars are actually in the queue? (Nobles are always telling the truth and liars are always lying.)

**A)** 0 **B)** 12 **C)** 13 **D)** 24 **E)** it cannot be determined

16) If a	b = ab + a + b and 3	5 = 2	x, then x equals	
<b>A</b> ) 3	<b>B</b> ) 6	<b>C</b> ) 7	<b>D</b> ) 10	<b>E</b> ) 1

17) The centres of the four illustrated circles are in the corners of the square. The two big circles touch each other and also the two little circles. With which factor do you have to multiply the radii of the little circles to obtain the radius of the big circles?

**A**)  $\frac{2}{9}$  **B**)  $\sqrt{5}$  **C**) 0,8· $\pi$  **D**) 2,5

18) The difference of  $\sqrt{n}$  and 10 is less than 1. How many whole numbers n are there with this characteristic?

19) Friday writes different positive whole number that are all less than 11 next to each other in the sand. Robinson Crusoe looks at the sequence and notices with amusement that adjacent numbers are always divisible by each other. What is the maximum amount of numbers he could possibly have written in the sand?

20) There are three great circles on a sphere that intersect each other in a right angle. Starting in point S a little bug moves along the great circles in the direction indicated. At crossings it turns alternately to the right or left. How many quarter circles does it crawl along until it is back in point S? A) 6 B) 9 C) 12 D) 15 E) 18



## - 5 Points Questions -

21) How many 0s have to replace the Star \* in the decimal number 1.\*1 to obtain a number that is smaller than  $\frac{2009}{2008}$ , but bigger than  $\frac{20009}{20008}$ ? A) 1 B) 2 C) 3 D) 4 E) 5

22) How many 10-digit numbers exist that are solely made up of the numbers 1, 2 and 3 (but not necessary all of them) and where adjacent numbers always differ by exactly 1?

**A**) 16 **B**) 32 **C**) 64 **D**) 80 **E**) 100

23) If 
$$a = 2^{25}$$
,  $b = 8^8$  and  $c = 3^{11}$  then  
**A**)  $a < b < c$ 
**B**)  $b < a < c$ 
**C**)  $b < c < a$ 
**D**)  $c < a < b$ 
**E**)  $c < b < a$ 

E) 12

E)

24) All factors of a number N (with the exception of 1 and N itself) are written down one after the other. It turns out that the biggest factor is 45 times as big as the smallest number. For how many numbers N is that true?

A) 0 B) 1 C) 2 D) more than 2 E) it cannot be determined

25) Kangoo has 2009 unit dice that he is putting together to a big cuboid. He has also got 2009 square unit sticker with the measurements  $1 \times 1$  with which he tries to paste up the surface area of the cuboid. He manages to do that and even has got some spare stickers. How many are there left over?

**A**) more than 1000 **B**) 763 **C**) 476 **D**) 49 **E**) it is not achievable

26) Robert wants to place stones on a  $4 \times 4$  gameboard so that the number of stones in each row and column is different; i.e. there are 8 different amounts. To achieve this he can place one or several stones in any one field or even leave single fields empty. What is the minimum number of stones needed to do this?

**A**) 14 **B**) 16 **C**) 20 **D**) 24 **E**) 25

27) A number of oranges, peaches, apples and bananas are put down in a row. What is the minimum number of fruits needed to lay them down so that each fruit lies next to each other fruit at least once in that row?

**A**) 4 **B**) 5 **C**) 8 **D**) 11 **E**) this is not achievable

28) What is the smallest whole number n for which the expression  $(2^2-1)\cdot(3^2-1)\cdot(4^2-1)\cdot i \cdot (n^2-1)$  is a square number?

**A**) 6 **B**) 8 **C**) 16 **D**) 27 **E**) a different number

29) A kangaroo is sitting in the origin of a kartesian co-ordinate system. With each bounce it can jump one unit in the horizontal or vertical direction. How many points are there, where the kangaroo could be after 10 jumps?

**A**) 121 **B**) 100 **C**) 400 **D**) 441 **E**) a different number

30) A single digit prime number is called šstrangeõ. A prime number with more than one digit is called šstrangeö if the numbers that are obtained by cancelling the first and the last digit respectively are both strange prime numbers again. How many strange prime numbers are there?

**A**) 6 **B**) 7 **C**) 8 **D**) 9 **E**) 11