Olympiad Problems 2010-2011 Division M

WITH ANSWERS AND SOLUTIONS

MATH OLYMPIADS
Since 1979

Our Thirty-SecondYear

Mathematical Olympiads for Elementary and Middle Schools
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1A **Time: 3 minutes**
25 digits are shown.
Find the sum of the digits.

		2	2	2	2
		2	2	6	2
		2	6	6	2
		6	6	6	2
	4	4	4	4	
4	4	4	4	4	

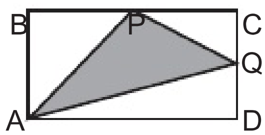
1B **Time: 4 minutes**
How many quarters (worth 25 cents each) must be added to 12 nickels (worth 5 cents each), so that the average value of a coin in the new enlarged collection is 10 cents?

1C **Time: 5 minutes**
How many different sums can be obtained by adding two <u>different</u> integers chosen from the set below?

$$\{-12, -11, -10, \ldots, +6, +7, +8\}$$

1D **Time: 5 minutes**
561 is the product of 3 different prime numbers. How many factors of 561 are not prime?

1E **Time: 7 minutes**
In rectangle ABCD, P is the midpoint of side $\overline{BC}$ and Q is the midpoint of CD. The area of ΔAPQ is what fractional part of the area of rectangle ABCD?



2A **Time: 3 minutes**
Write as a single decimal:

$$1-\frac{2}{10}+\frac{3}{100}-\frac{4}{1000}$$

2B **Time: 4 minutes**
Amy picks a whole number, squares it and then subtracts 1. She gives her final number to Brian. Brian adds 3 to the number Amy gave him and then doubles that result.

Brian's final result is 54. With what number did Amy start?

2C **Time: 6 minutes**
Alex, Bruno, and Charles each add the lengths of two sides of the same triangle correctly. They get 27 cm, 35 cm, and 32 cm, respectively. Find the perimeter of the triangle, in cm.

2D **Time: 6 minutes**
The first three terms in a sequence are: 1, 2, 3. Each term after that is the opposite of the sum of the three previous terms. For example, the 4^{th} term is $^{-}6$ (the opposite of 1+2+3), and the 5^{th} term is 1. What is the 99^{th} term?

2E **Time: 7 minutes**
Find the whole number value of

$$\sqrt{1+3+5+...+45+47+49}$$

3A **Time: 4 minutes**

When Isaac opens a book, the product of the page numbers on the open pages is 420. Find the sum of the two page numbers.

3B **Time: 5 minutes**

A squirrel buries a total of 80 acorns in N holes. Find the greatest possible value of N, provided:

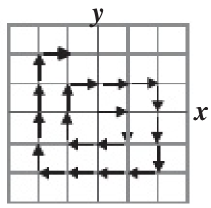
(1) No hole is empty, and
(2) No two holes contain the same number of acorns.

3C **Time: 5 minutes**

The sum of a proper fraction in lowest terms and its reciprocal equals $2\frac{4}{15}$. Find the original proper fraction.

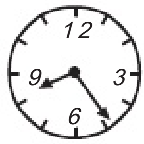
3D **Time: 7 minutes**

The picture shows a "spiral" that begins at the origin (0,0) and passes through every lattice point in the plane. Each small arrow is 1 unit in length. Following the "spiral", what is the length of the path from the origin to the point (5,3)?



3E **Time: 7 minutes**

How many degrees are in the angle formed by the hands of a clock at 8:24?



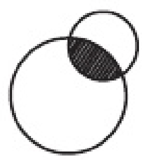
4A **Time: 4 minutes**
Some students are in a line. Abby is in the center of the line. Sara is 3 places in front of her, Eli is 4 places behind Sara, and Kayla is 2 places in front of Eli. Kayla is the third person in line. How many students are in the line?

4B **Time: 4 minutes**
Given the data: 3, 6, 6, 8, 10, 12.

Express in lowest terms: $\dfrac{3 \times \text{median} - \text{mode}}{6 \times \text{mean}}$

4C **Time: 4 minutes**
Find the integer that exceeds $^{-}5$ by the same amount that $^{+}13$ exceeds $^{-}1$.

4D **Time: 6 minutes**
A circle with radius 5 cm intersects a circle with radius 3 cm as shown. The area of the shaded region is $\frac{7\pi}{2}$ square cm. Find the total combined area <u>inside</u> the circles, but <u>outside</u> the shaded region. Leave your answer in terms of π.



4E **Time: 8 minutes**
How many different triangles can be formed whose 3 vertices are chosen from the rectangular array of 8 points shown?

5A **Time: 4 minutes**
How many 2-digit numbers are there in which the ones digit is greater than the tens digit?

5B **Time: 5 minutes**
A bank has two plans for checking accounts. In plan A, the charge is \$7.50 a month with no fee for each check. In plan B, the charge is \$3 a month plus an additional 20 cents for each check written. What is the ~~least~~ number of checks a customer must write each month so that plan A costs less than plan B?

5C **Time: 5 minutes**
Suppose the base of a triangle is increased by 20%, and its height is increased by 30%. By what percent is the area of the triangle increased?

5D **Time: 6 minutes**
Starting with 1, Sara lists the counting numbers in order but omits all those that use the digit 9. What is the 300^{th} number on her list?

5E **Time: 8 minutes**

Line segments form a path that starts at (0,0), is drawn to (1,0), and then to (1,2). Each new segment forms a right angle with the segment before it and is 1 unit longer than that segment. The path ends at (0,0). How many segments are in the shortest possible path?
(*Hint: Consider horizontal and vertical segments separately.*)

ANSWERS AND SOLUTIONS

Note: Number in parentheses indicates percent of all competitors with a correct answer.

OLYMPIAD 1 — NOVEMBER 17, 2010

Answers: [1A] 92 [1B] 4 [1C] 39 [1D] 5 [1E] 3/8

69% correct

1A METHOD 1: *Strategy: Count the number of times each digit appears.*
Multiply each value by the number of times it appears. Add those products.
$10\times2 + 9\times4 + 6\times6 = 92$. The sum is 92.

METHOD 2: *Strategy: Separate into a rectangle of 2s and a triangle of 4s.*

```
   2 2 2 2        2 2 2 2
   2 2 6 2        2 2 2 2               4
   2 6 6 2        2 2 2 2             4 4
   6 6 6 2   =    2 2 2 2     +     4 4 4
 4 4 4 4                          4 4 4 4
4 4 4 4 4                       4 4 4 4 4
```

The sum is $16 \times 2 + 15 \times 4 = 92$.

58%

1B METHOD 1: *Strategy: Compare each coin to the average value.*
Each nickel is worth 5 cents less than the average; each quarter is worth 15 cents more. Then each quarter, combined with 3 nickels, has an average value of 10 cents. There are 12 nickels, so 4 quarters must be added to the collection.

METHOD 2: *Strategy: Use algebra.*
Let Q represent the number of quarters to be added. The total value of the coins is 25Q + 60. The total number of coins is Q + 12. Then $\frac{25Q+60}{Q+12} = 10$. Solving, Q = 4. Thus, 4 quarters must be added.

FOLLOW-UP: *How many \$5 bills must be added to twenty \$100 bills so that the average value of all the bills is \$10?* [360]

10%

1C *Strategy: Find the least and greatest possible sums.*

The least possible sum is obtained by adding –12 and –11. The greatest possible sum is obtained by adding 7 and 8. Every integer between these extremes is also a possible sum. By examining a number line from –23 to +15 inclusive, you should see 23 negative sums, 15 positive sums, and zero. There are 39 different sums that can be obtained.

FOLLOW-UP: *How many distinct sums can be obtained by adding two different integers chosen from the consecutive even integers from -12 to +8, inclusive?* [19]

10%

1D *METHOD 1: Strategy: Use the divisibility rules to find factors.*
The sum of the digits of 561 is 12, so 3 is a factor and $561 \div 3 = 187$. Then 187 satisfies the test of divisibility for multiples of 11 (that is, $1 - 8 + 7 = 0$), so 11 is also a factor and $187 \div 11 = 17$. Thus 561 factors into $3 \times 11 \times 17$. The table shows all factor pairs of 561. Of the 8 factors, 3 are prime. Therefore, there are 5 factors of 561 which are not prime.

561	
1	561
3	187
11	51
17	33

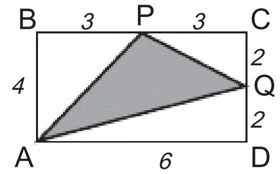
METHOD 2: Strategy: Find the total number of factors without factoring.
Call the prime factors P, Q, and R. Their product is PQR and the table shows all its factor pairs. In all, 3 of the 8 factors are prime. Thus, there are 5 factors of 561 which are not prime.

PQR	
1	PQR
P	QR
Q	PR
R	PQ

FOLLOW-UPS: *(1) N is the product of 4 different prime numbers. How many factors of N are not prime?* [12] *(2) N is the product of 5 different prime numbers. How many factors does it have altogether?* [32] *(3) What is the least number with exactly 8 factors?* [30] *(4) Why is 1 neither prime nor composite?* [A prime has exactly 2 factors and a composite has at least 3 factors. 1 has only one factor.]

19%

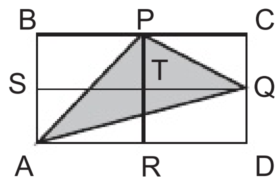
1E *Strategy: Find the unshaded area.*
METHOD 1: Strategy: Assign convenient numerical lengths to the sides.



Suppose for example that BC = 6 and CD = 4. Then BP and PC each are 3, and CQ and QD each are 2, and the area of rectangle ABCD is 24.

The area of ΔABP is $\frac{1}{2} \cdot 4 \cdot 3 = 6$, of ΔPCQ is $\frac{1}{2} \cdot 3 \cdot 2 = 3$, and of ΔQDA is $\frac{1}{2} \cdot 6 \cdot 2 = 6$. The total unshaded area is $6 + 3 + 6 = 15$, and the area of ΔAPQ is 9. The area of ΔAPQ is $\frac{9}{24} = \frac{3}{8}$ of the area of rectangle ABCD.

METHOD 2: Strategy: Split the figure into more convenient shapes.



Draw $\overline{PR}$ and $\overline{QS}$ parallel to the sides of the rectangle as shown.

The area of ΔABP is $\frac{1}{2}$ the area of rectangle ABPR, which is $\frac{1}{2}$ the area of ABCD. Then ΔABP is $\frac{1}{4}$ the the area of ABCD.

The area of ΔQDA is $\frac{1}{2}$ the area of rectangle SQDA which is $\frac{1}{2}$ the area of ABCD. So ΔQDA is $\frac{1}{4}$ the area of ABCD.

The area of ΔPCQ is $\frac{1}{2}$ the area of rectangle PCQT which is $\frac{1}{4}$ the area of ABCD. So ΔPCQ is $\frac{1}{8}$ the area of ABCD.

The unshaded area then is $\frac{1}{4} + \frac{1}{4} + \frac{1}{8} = \frac{5}{8}$ of the area of ABCD.

Thus, the area of ΔAPQ is $\frac{3}{8}$ of the area of rectangle ABCD.

Answers: [2A] .826 [2B] 5 [2C] 47 [2D] 3 [2E] 25

53% correct

2A METHOD 1: *Strategy: Add decimals in a convenient order.*
Rewrite each term as a decimal and then combine terms of the same sign.

$$1-\frac{2}{10}+\frac{3}{100}-\frac{4}{1000} = 1-.2+.03-.004$$
$$= (1+.03) - (.2+.004)$$
$$= 1.03 - .204$$
$$= .826$$

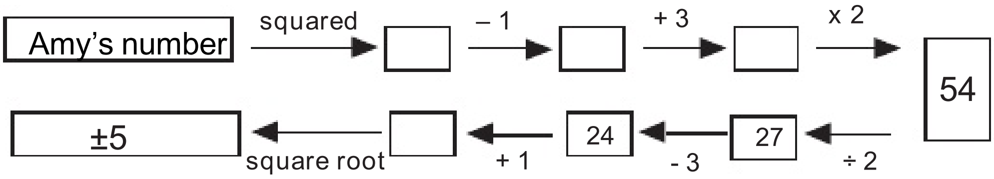
METHOD 2: *Strategy: Eliminate the denominators (temporarily).*
Multiply each term by 1000 and reduce. Then divide by 1000 to undo step 1.

$$1000-\frac{2000}{10}+\frac{3000}{100}-\frac{4000}{1000}$$
$$= 1000 - 200 + 30 - 4$$
$$= 826$$

Now divide by 1000 to undo the first step. The result is 0.826.

78%

2B *Strategy: Work backwards.*



Amy picked a whole number, so she started with 5.

47%

2C METHOD 1: *Strategy: Use the symmetry of the given information.*
27, 35, and 32 are each the sum of a different pair of sides of the triangle. Then 27 + 35 + 32 is the sum of the three sides, each counted twice. Thus 94 is twice the perimeter and the perimeter of the triangle is 47 cm.

METHOD 2: *Strategy: Use algebra to find the length of each side.*
Let a, b, and c represent the sides of the triangle. Then the three equations are:
(1) a + b = 27, (2) a + c = 35, and (3) b + c = 32.
One way to solve this system is to add equations (1) and (2), and then subtract (3) from the result.

(1) $a + b = 27$
(2) $a + c = 35$
$2a + b + c = 62$
(3) $b + c = 32$
$2a = 30$
(4) $a = 15$

Now add (4) and (3):
(4) $a = 15$
(3) $b + c = 32$
$a + b + c = 47$

The perimeter of the triangle is 47 cm.

FOLLOW-UPS: *(1) Suppose the 3 numbers given in the problem are each the sum of 2 sides of a parallelogram. What is the perimeter of the parallelogram?* [62 2/3 cm] *(2) The area of 3 different faces of a box (rectangular solid) are 20, 28, and 35 sq cm. What is the volume of the box?* [140 cm^3]

38%

2D *Strategy: Continue the sequence and look for a pattern.*
The sixth term is $^-(3 + {}^-6 + 1) = {}^-({}^-2) = 2$. The sequence is 1, 2, 3, -6, 1, 2, 3, -6, 1, 2, 3, -6, and so on. The terms repeat in groups of four. Thus, every fourth term is $^-6$. Therefore the 100th term is -6, and so the 99th term is 3.

FOLLOW-UP: *A sequence begins 1, 2, 3, … The fourth term is the third less the second; the fifth term is the fourth less the third, and so on. What is the 49th term?* [-1]

41%

2E METHOD 1: *Strategy: Find the sum of the numbers.*
In the series 1 + 3 + 5 … + 49, we get from 1 to 49 by adding 2 twenty four times, so the series has 25 terms. Pair these terms as follows, working from the outside inward (1 + 49) + (3 + 47) + (5 + 45), and so on. The sum of each pair is 50 and there are 12 pairs. The number without a pair is the middle one, 25. The sum is then 12 × 50 + 25 = 625. Then, $\sqrt{625}$ is 25.

METHOD 2: *Strategy: Look for a pattern in the partial sums.*
The table below examines the square root of the sums of the first few terms. In each case, the square root is equal to the number of terms added.

Number of Terms	Sum of Terms	Square Root of the Sum of Terms
1	1	1
2	4	2
3	9	3
4	16	4
5	25	5

There are 25 terms in the given sequence (25 odd numbers from 1 to 49). Therefore the square root of the sum of the series is 25.

OLYMPIAD 3 JANUARY 12, 2011

Answers: [3A] 41 [3B] 12 [3C] 3/5 [3D] 82 [3E] 108

66% correct

3A METHOD 1: *Strategy: Look for a perfect square near 420.*

The page numbers differ by 1, so the factors of 420 must be nearly equal. $20^2 = 400$, so try 20 as a factor. 420 = 20 × 21, and the sum of the two page numbers is 41.

METHOD 2: *Strategy: Combine the prime factors of 420.*
The prime factorization of 420 is 2 × 2 × 3 × 5 × 7. Combining the factors into two products whose difference is 1, we get 20 × 21. Their sum is 41.

FOLLOW-UP: *The sum of the 6 page numbers in a chapter of the book is 153. What is the number on the first page of the chapter?* [23]

52%

3 B *Strategy: Minimize the number of acorns in each hole.*
Each hole requires a different number of acorns, so put 1 in the first hole, 2 in the second, and so on. Continue until the total number of acorns is near 80. $1 + 2 + 3 + \ldots + 11 + 12 = 78$, and $1 + 2 + 3 + \ldots + 13 = 91$. 13 holes require at least 91 acorns. The squirrel puts the first 78 acorns into 12 holes as indicated. The squirrel then can put the other two acorns into the 12th hole, making 14 acorns in that hole. The greatest possible value of N is 12.

FOLLOW-UPS: *(1) Aside from 1,2,3, . . .,11,14 above, how many other sets of 12 different counting numbers have a sum of 80?* [1: the last 2 numbers are 12 and 13] *(2) In how many different ways can the squirrel bury 20 acorns in two holes if each hole has a different number of acorns and no hole is empty?* [9] *(3) 20 acorns in 3 holes?* [24]

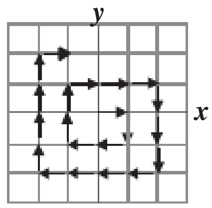
29%

3 C *Strategy: List the possible numerators and denominators.*
Denote the fraction and its reciprocal by $\frac{A}{B}$ and $\frac{B}{A}$. Their sum has a denominator of 15, so A and B must be chosen from {1, 3, 5, 15}. Neither A nor B can be 15, for $\frac{15}{1}$, $\frac{15}{3}$, and $\frac{15}{5}$ are each larger than $2\frac{4}{15}$. Likewise, neither A nor B can be 1, for $\frac{5}{1}$ and $\frac{3}{1}$ are also larger than $2\frac{4}{15}$. The only possibility is that the fractions are $\frac{3}{5}$ and $\frac{5}{3}$. In fact, their sum is $\frac{34}{15} = 2\frac{4}{15}$. Of these two, the proper fraction is $\frac{3}{5}$.

FOLLOW-UP: *What is the least possible sum of a positive fraction (not necessarily proper) and its reciprocal?* [2]

22%

3 D *Strategy: Look for a pattern.*
Consider the points where the path changes direction. Since (5,3) is in the first quadrant, find the path length to each of the upper right "corners".



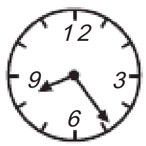
Coordinates of corner point	Path length
(1,0)	1
(2,1)	1 + 1 + 2 + 2 + 3 = 9
(3,2)	9 + 3 + 4 + 4 + 5 = 25

The path lengths are consecutive odd squares. In fact, they are the squares of the sum of the coordinates of the corner points. The corner point closest to (5,3) is (5,4) and the path length to (5,4) is 81 units. Following the spiral, (5,3) is the next lattice point reached, so the length of the path is 82 units.

FOLLOW-UP: *In the other 3 quadrants what patterns are formed by the path's lengths to the corners?*

14%

3 E METHOD 1: *Strategy: Find the speed of each hand in degrees per minute.*
The minute hand moves 360° in 1 hour and thus moves 6° per minute. The hour hand moves $\frac{1}{12}$ as far as the minute hand every hour and therefore moves $\frac{1}{12}$ as far every minute, i.e. $\frac{1}{2}$° per minute. At 8:00 the hour hand is 240° ahead of the minute hand. In the next 24 minutes the hour hand moves an additional 12° and the minute hand moves 144°. At 8:24 the angle between the hands is 240 + 12 – 144 = 108°.



METHOD 2 on next page

3E METHOD 2: *Strategy: Start at 12:00 and see how far each hand has rotated.*
From 8:00 to 8:24, the minute hand has rotated $\frac{24}{60} = \frac{2}{5}$ of the way around the clock. That is, it has rotated $\frac{2}{5}$ of 360º = 144º. Think of 12:00 as 0º, 3:00 at 90º, and 6:00 as 180º. Then the minute hand is pointing to 144º.

Meanwhile, the hour hand, which needs 12 hours to rotate 360º, rotates 30º every hour. Thus, at 8:00, it was pointing to 240º. At 8:24, it has rotated another $\frac{2}{5}$ of 30º = 12º and is pointing to 252º. At 8:24 the angle between the hands is 252 – 144 = 108º.

OLYMPIAD 4 — FEBRUARY 9, 2011

Answers: [4A] 7 [4B] 1/3 [4C] 9 [4D] 27π [4E] 48

53% correct

4A METHOD 1: *Strategy: Start with Kayla, whose position is known.*

Kayla is the third person.	___ , ___ , K , …
Kayla is 2 places in front of Eli.	___ , ___ , K , ___ , E , …
Eli is 4 places behind Sara.	S , ___ , K , ___ , E , …
Sara is 3 places in front of Abby.	S , ___ , K , A , E , …
Abby is in the center of the line.	S , ___ , K , A , E , ___ , ___

Abby is the 4th person in line. There are 3 people in front of her and 3 people behind her. There are 7 students in the line.

METHOD 2: *Strategy: Create the line, keeping Abby in the center.*

Sara is 3 places in front of Abby.	…, S , ___ , ___ , A , ___ , ___ , ___
Eli is 4 places behind Sara.	…, S , ___ , ___ , A , E , ___ , ___
Kayla is 2 places in front of Eli.	…, S , ___ , K , A , E , ___ , ___

Kayla is the 3rd person in line, so no additional spaces are needed in front of Sara. There are 7 students in the line.

45%

4B METHOD 1: *Strategy: Calculate the required values.*

The median is $\frac{8+6}{2} = 7$. The mode is 6. The mean is $\frac{3+6+6+8+10+12}{6} = \frac{45}{6}$.

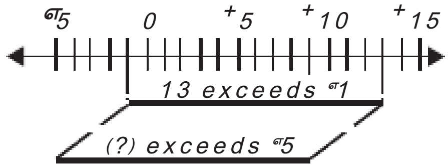
Then $\frac{3\times7-6}{6\times\left(\frac{45}{6}\right)} = \frac{15}{45} = \frac{1}{3}$.

FOLLOW-UPS: (1) The mean and median of a set of five different positive integers is 12. One number is 3 less than the median and another is half the mean. What is the greatest possible integer in the set? [20] (2) The mean, median and mode of a set of five positive integers are all equal. Three of the numbers are 9, 13, and 41. Find the missing numbers that satisfy this condition. [21, 21] and [41, 101]

61%

4C *METHOD 1: <u>Strategy</u>: Do the arithmetic.*
13 exceeds $^{-}1$ by $13 - {}^{-}1 = 14$. The integer that exceeds $^{-}5$ by 14 is $^{-}5 + 14 = 9$.

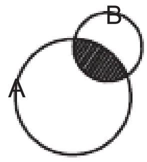
METHOD 2: <u>Strategy</u>: Draw a number line.
The upper line segment shows the amount by which 13 exceeds $^{-}1$. The lower line segment shows the amount by which the desired number exceeds $^{-}5$.



Sliding the upper segment into the lower position requires moving the left endpoint from $^{-}1$ to $^{-}5$, 4 units to the left. Then the right endpoint also slides 4 units to the left, from 13 to 9. The integer is 9.

16%

4D *METHOD 1: <u>Strategy</u>: Find the unshaded area inside each circle.*
The areas inside the two circles are 25π and 9π respectively.



The unshaded area inside circle A is $25\pi - \frac{7\pi}{2} = \frac{43\pi}{2}$.

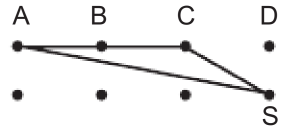
The unshaded area inside circle B is $9\pi - \frac{7\pi}{2} = \frac{11\pi}{2}$.

The total unshaded area is $\frac{43+11}{2}\pi = 27\pi$ sq cm.

METHOD 2: <u>Strategy</u>: Start with the total area of the two circles.
The sum of the areas of the circles is $25\pi + 9\pi = 34\pi$. This, however, counts the shaded area twice, once for each circle. The total unshaded area is $34\pi - 2(\frac{7\pi}{2}) = 27\pi$.

7%

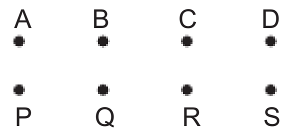
4E *METHOD 1: <u>Strategy</u>: Count in an organized way.*
To form a triangle, two points must be chosen from 1 row and one from the other. Suppose two points are chosen from the top row. Label the four points in the top row A, B, C, D. Two points may be chosen in 6 ways (AB, AC, AD, BC, BD, CD). For each of these 6 pairs of points, the third vertex may be any of the 4 points in the bottom row. There are then 6 × 4 = 24 triangles using 2 points from the top row and 1 from the bottom row.



ΔACS is shown.

Likewise, there are 24 triangles using 2 points from the bottom row and 1 from the top. In all, 48 triangles can be formed

METHOD 2: <u>Strategy</u>: Make an organized list.
Label all 8 points, as shown. List all 18 triangles containing vertex A, all 14 triangles containing vertex B but not A, all 10 triangles containing vertex C but not A or B, and all 6 triangles containing vertex D but not A or B or C. A total of 48 triangles can be formed..



ΔABP	ΔACP	ΔADP
ΔABQ	ΔACQ	ΔADQ
ΔABR	ΔACR	ΔADR
ΔABS	ΔACS	ΔADS
ΔAPQ	ΔAPR	ΔAPS
ΔAQR	ΔAQS	ΔARS

ΔBCP	ΔBDP
ΔBCQ	ΔBDQ
ΔBCR	ΔBDR
ΔBCS	ΔBDS
ΔBPQ	ΔBQR
ΔBPR	ΔBQS
ΔBPS	ΔBRS

ΔCDP	ΔCPQ
ΔCDQ	ΔCPR
ΔCDR	ΔCPS
ΔCDS	ΔCQR
ΔCQS	ΔCRS

ΔDPQ
ΔDPR
ΔDPS
ΔDQR
ΔDQS
ΔDRS

METHOD 3 on next page

4E *METHOD 3: Strategy: Use combinations. Subtract the triples that don't work.*
Given eight points, three points may be chosen in ${}_8C_3 = 56$ ways.

No triangle is formed if all three points are chosen from the same row. There are ${}_4C_3 = 4$ sets of three points in the top row and another 4 sets in the bottom row.

Thus 56 – 4 – 4 = 48 triangles can be formed.

FOLLOW-UP: *How many triangles can be formed with vertices chosen from a 3x3 array of points?* [76]

OLYMPIAD 5 — MARCH 9, 2011

Answers: [5A] 36 [5B] 23 [5C] 56 [5D] 363 [5E] 7

60% correct

5A *METHOD 1: Strategy: Group the numbers by tens and look for a pattern.*

Interval	Satisfactory Numbers	Number of Numbers
10 - 19	12, 13, 14, 15, …, 19	8
20 - 29	23, 24, 25, …, 29	7
30 - 39	34, 35, …, 39	6
⋮	⋮	⋮
80 - 89	89	1
	Total	36

There are 36 two-digit numbers in which the ones digit is greater than the tens digit.

METHOD 2: Strategy: Eliminate all unwanted numbers.
There are 90 two-digit counting numbers. Eliminate the 9 numbers with equal digits (11, 22, etc.). Then eliminate the 9 multiples of 10. The remaining 72 numbers have 2 unequal nonzero digits. In half of them the ones digit exceeds the tens digit. There are then 36 such numbers.

63%

5B *METHOD 1: Strategy: Make a table.*

Number of Checks	A: Flat Fee = $7.50	B: $3 + 20 cents / check
1	$7.50	$3.20
10	$7.50	$5.00
20	$7.50	$7.00
21	$7.50	$7.20
22	$7.50	$7.40
23	$7.50	$7.60

Therefore, the least number of checks such that Plan A costs less than Plan B is 23.

METHOD 2: Strategy: Use Algebra.
Let N represent the number of checks for which plan A is cheaper.

$$20N + 300 > 750 \quad \text{(cost, in cents to avoid decimals)}$$
$$20N > 450$$
$$N > 22\tfrac{1}{2}$$

At least 23 checks are needed for Plan A to cost less than Plan B.

27%

5C METHOD 1: Strategy: *Assign arbitrary dimensions.*

In ratio problems, assigning convenient measures will not affect the answer. For ease of computation, let the base be 20 and the height 10 so that the area is $\frac{1}{2}(20)(10) = 100$ sq units. If the base is increased by 20%, the new base is 24. If the height is increased by 30%, the new height is 13. The area of the new triangle is $\frac{1}{2}(24)(13)=156$ square units. The increase over the original 100 is 56. Therefore, the area is increased by 56 compared to 100 = 56%.

METHOD 2: Strategy: *Compare areas using the area formula.*

The original area of the triangle is given by $A = \frac{1}{2}bh$. The changed area of the triangle is given by $\frac{1}{2}(1.20b)(1.30h) = \frac{1}{2}(1.56)bh$, which in turn equals 1.56 times $\frac{1}{2}bh$. Therefore the area of the new triangle is 1.56 times as great as the area of the original. This is 56% greater than the area of the original. The increase is 56%.

FOLLOW-UP: *Suppose the base of a triangle is decreased by 20%, and its height is decreased by 30%. By what percent is the area of the triangle decreased?* [44]

14%

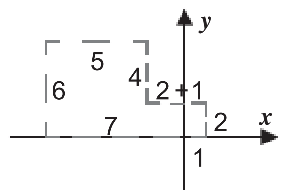
5D Strategy: *Determine how many of the first 300 numbers can't be used.*

In each of the sets 1 through 100, 101 through 200, and 201 through 300, 10 numbers have a ones digit of 9, and 10 numbers have a tens digit of 9. However, in each set the number ending in 99 has been counted twice, so each set has 19 numbers that contain a digit of 9. In the overall list 1 through 300, 57 numbers must be eliminated. Refill the list with the next 57 numbers, 301 through 357. There are 5 numbers in this set that can't be used (309, 319, 329, 339, 349). Add on 5 more numbers, 358 through 362. One of these, 359, can't be used, so add on one more number. The 300th number on Sara's list is 363.

12%

5E Strategy: *Draw possible paths.*

By drawing some paths, you may see two things: (1) the path can turn either left or right, and (2) each horizontal segment has an odd length while each vertical segment has an even length.



Consider first the horizontal segments that can end back at the starting point. Neither {1 unit and 3 units} nor {1, 3 and 5 units} can end at the origin, but {1, 3, 5, and 7 units} can. By traveling 1 and 7 units to the right, and 3 and 5 units to the left, the horizontal part of the path can end at zero.

Now consider 6 and 8, the even numbers on either side of 7. Since 2 + 4 = 6, if 2 and 4 are directed up and 6 is directed down, the vertical part of the path also ends at zero. Thus the shortest possible path consists of 7 segments and is shown above.

FOLLOW-UPS: *(1) A path consisting of* N *line segments is drawn in the coordinate plane. The first segment starts at (0,0) and is drawn to (2,0). The second segment starts at (2,0) and is drawn to (2,4). Each of the* N *segments is drawn at a right angle to the segment before it and is 2 units longer than that segment. The* Nth *segment ends at (0,0). What is the least possible value of* N? [7] *(2) What is the least possible value of* N *greater than 7?* [8] *(3) What are the next 2 possible values of* N? [15, 16]