MATHCOUNTS®

2016 Chapter Competition Solutions

Are you wondering how we could have possibly thought that a Mathlete® would be able to answer a particular Sprint Round problem without a calculator?

Are you wondering how we could have possibly thought that a Mathlete would be able to answer a particular Target Round problem in less 3 minutes?

Are you wondering how we could have possibly thought that a particular Team Round problem would be solved by a team of only four Mathletes?

The following pages provide solutions to the Sprint, Target and Team Rounds of the 2016 MATHCOUNTS® Chapter Competition. These solutions provide creative and concise ways of solving the problems from the competition.

There are certainly numerous other solutions that also lead to the correct answer, some even more creative and more concise!

We encourage you to find a variety of approaches to solving these fun and challenging MATHCOUNTS problems.

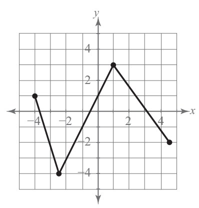
Special thanks to solutions author
Mady Bauer
for graciously and voluntarily sharing her solutions
with the MATHCOUNTS community
for many, many years!

2016 Chapter Competition

Sprint Round

1. *Given:* Count backwards from 155 by 4.
Find: The 9th number.
The first number is 155.
The second number is 151, or 4 less.
The ninth number is $4 \times 8 = 32$ less.
155 – 32 = 123 **Ans.**

2. *Given:* The graph, as displayed below.
Find: The difference between the maximum and minimum values.



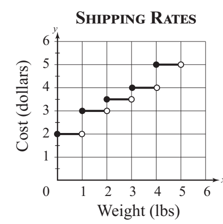
The highest point in terms of y is the third point from the right with coordinates (1, 3). The lowest point in terms of y is the second point from the right with coordinates (−2.5, −4) – at least it looks like around −2.5. The difference in the y-coordinates is $3 - (-4) = 7$ **Ans.**

3. *Given:* $\frac{1}{b} = \frac{b}{a}, b = -1$
Find: a
Substituting −1 for b, we get $1/{-1} = -1/a$.
Cross-multiplying, we find that
$a = (-1)(-1) = 1$ **Ans.**

4. *Find:* The sum of all two-digit multiples of 3 that have a units digit of 1.
21 is the first two-digit multiple of 3 which has a units digit of 1. Each successive two-digit multiple of 3 with a units digit of 1 is 30 larger. There are three in all: 21, 51 and 81. Their sum is
21 + 51 + 81 = 153 **Ans.**

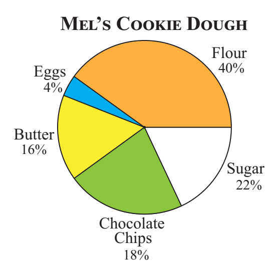
5. *Find:* the sum of the first 8 terms of the sequence beginning -4, 5 … where each term is the sum of the previous two terms.
The first eight terms of the sequence are −4, 5, 1, 6, 7, 13, 20 and 33. Their sum is
−4 + 5 + 1 + 6 + 7 + 13 + 20 + 33 = 81 **Ans.**

6. *Given:* The graph displayed below.
Find: the total cost to ship 3 packages weighing 1.8 lbs., 2 lbs. and 4.4 lbs.



It costs \$3 to ship the 1.8-lb. package, \$3.50 to ship the 2-lb. package and \$5 to ship the 4.4-lb. package. The total cost to mail all three packages is $3 + 3.50 + 5 = 11.50$ **Ans.**

7. *Given:* the graph below.
Find: the degree measure of the portion of the cookie that is flour.



The cookie is made of 40% flour and. 40% of 360° is $360 \times 0.4 = 144$ **Ans.**

8. *Given:* 3 zoguts and 4 gimuns costs \$18.
2 zoguts and 3 gimuns costs \$13.
Find: the cost of 1 zogut and 1 gimun
Let z = the cost of a zogut and g = the cost of a gimun. We can write the equations

$3z + 4g = 18$ (Eq. 1)
$2z + 3g = 13$ (Eq. 2)
Subtracting Eq. 2 from Eq. 1, we get
$1z + 1g = 18 - 13 = 5$ **Ans.**

9. *Given:* A rectangular piece of paper with a 40-inch length. It is folded in half. The ratio of the long side of the original sheet to the short side of the original sheet is the same as the ratio of the long side of the folded sheet to the short side of the folded sheet.
Find: the length of the short side of the original sheet.
Let s = the length of the short side of the original sheet, which is also the long side of the folded sheet. The length of the short side of the folded sheet is 40/2 = 20.
We have the ratio $\frac{40}{s} = \frac{s}{20}$. Cross-multiplying yields $s^2 = 800$. So the length of the short side of the original sheet is $s = \sqrt{800} = 10\sqrt{8} = 20\sqrt{2}$ **Ans.**

10. *Find:* the greatest multiple of 3 that can be formed using one or more of the digits 2, 4, 5 and 8, using each digit only once.
A number is a multiple of 3 if the sum of its digits is divisible by 3. Since 8 + 5 + 4 + 2 = 19, which is not divisible 3, we know that no number formed using all four digits will be a multiple of 3. The four combinations of three digits that can be used to form 3-digit numbers are 8, 5, 4; 8, 5, 2; 8, 4, 2; and 5, 4, 2. Since 8 + 5 + 4 = 17, 8 + 4 + 2 = 14 and 5 + 4 + 2 = 11, none of which is divisible by 3, we know that no 3-digit number formed with these combinations of digits will be a multiple of 3. But 8 + 5 + 2 = 15, which is divisible by 3. The greatest multiple of 3 formed using the given numbers is 852 **Ans.**

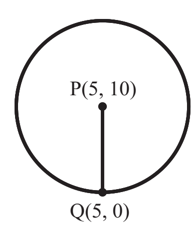
11. *Given:* $\frac{a^6b}{a^3b^5} = \frac{a^m}{b^n}$
Find: $m + n$
Simplifying the expression, we get
$$\frac{a^6b}{a^3b^5} = \frac{a^3}{b^4}$$
So $m = 3$, and $n = 4$. Therefore,
$m + n = 3 + 4 = 7$ **Ans.**

12. *Given:* 8% of x% of 200 = 4
Find: x
Rewriting the percents as fractions, the problem statement can be expressed algebraically as
$$\frac{8}{100}\left(\frac{x}{100} \times 200\right) = 4$$
Simplifying and solving for x, we get
$16x = 400$ and $x = 25$ **Ans.**

13. *Given:* There are 192 9th-graders and 136 10th-graders entered in a drawing.
Find: the probability a 10th-grader wins the drawing.
The probability is $\frac{136}{192+136} = \frac{136}{328} = \frac{17}{41}$ **Ans.**

14. *Given:* $2^2 \cdot 4^4 = 2^k$
Find: k
$2^k = 2^2 \cdot 4^4 = 2^2 \cdot (2^2)^4 = 2^2 \cdot 2^8 = 2^{10}$.
Therefore, $k = 10$ **Ans.**

15. *Given:* A circle with center P(5, 10) intersects the x-axis at Q(5, 0).
Find: the area of the circle.



The radius of the circle drawn from P to Q, as shown, has length. This is the radius 10 – 0 = 10. The area is $\pi r^2 = 100\pi$ **Ans.**

16. *Given:* Subtract 3 from 2*x* and divide the difference by 5. The result is 7.
Find: x
The problem statement can be expressed algebraically as $\frac{2x-3}{5} = 7$. Solving for *x*, we see that $2x - 3 = 35$ and $2x = 38$. So, $x = 19$ **Ans.**

17. *Given:* Melina's ratio of 2-point shot attempts to 3-point shot attempts is 4:1.
Find: the percent of Melina's attempted shots that are 3-point shots.
Of every 5 shot attempts, 1 is a 3-point shots and 4 are 2-point shots. So, the 3-point shots account for $\frac{1}{5} = 20\%$ **Ans.**

18. *Find:* the sum of the distinct prime factors of 2016.
The prime factorization of 2016 is $2^5 \times 7 \times 3^2$. The distinct prime factors 2, 3 and 7 have the sum $2 + 3 + 7 = 12$ **Ans.**

19. *Given:* The opposite faces of a six-sided die add up to 7. Two identical six-sided dice are placed as shown.
Find: the sum of the number of dots on the two faces that touch each other.

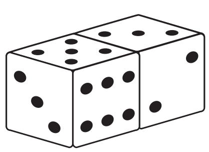


Figure 1

Since the dots on opposite faces of a die add to 7, it follows that the opposing sides must be 6 and 1, 5 and 2, 4 and 3. As Figure 1 shows, the first die is positioned so that the top face has 5 dots and the bottom face has 2 dots. The right face has 6 dots, and the left face has 1 dot. The front face has 3 dots, and the back face, which touches the front face of the second die, has 4 dots.

Now let's try to rotate the first die so it is positioned exactly like the second die so that corresponding sides on the dice have the same number of dots and are aligned the same way. If we rotate the first die so that the front face is now positioned on the top, we obtain the arrangement shown in Figure 2.

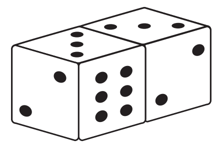


Figure 2

If we rotate the first die again so that the current front face is now on the right, we get the arrangement in Figure 3.

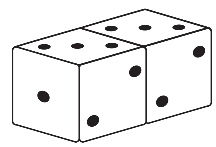


Figure 3

Now that all corresponding faces on the dice are the same, we can see that on the second die, the front face, which touches the back face of the first die, has 1 dot. The sum of these two values, then, is $4 + 1 = 5$ **Ans.**

20. *Find:* how many sets of two or three distinct positive integers have a sum of 8?
There are 3 sets of two integers that have a sum of 8: {1, 7}, {2, 6}, {3, 5}. The pair {4, 4} doesn't make it because the values aren't distinct. There are 2 sets of three integers that have a sum of 8: {1, 2, 5}, {1, 3, 4}. That makes the total number of sets $3 + 2 = 5$ **Ans.**

21. *Given:* $\frac{a+b}{2} = 3, \frac{b+c}{2} = 4, \frac{a+c}{2} = 5$

Find: $a + b + c$

The three given equations can be rewritten as

$a + b = 6$

$b + c = 8$

$a + c = 10$.

Adding these three equations yields $2a + 2b + 2c = 24$. Dividing both sides by 2, we get $a + b + c = 12$ **<u>Ans.</u>**

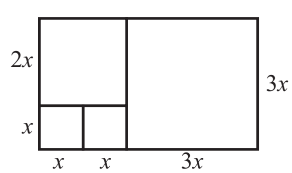
22. *Given:* The mean of a set of 5 numbers is $3k$. Add a sixth number to the set and the mean increases by k.

Find: the ratio of the sixth number to the sum of the first 5 numbers.

Let S be the sum of the first five numbers. Then we have $\frac{S}{5} = 3k$ and $S = 15k$. Let x be the sixth number. Then $\frac{S+x}{6} = 3k + k$ and $S + x = 24k$. Substituting $15k$ for S in $S + x = 24k$ yields $15k + x = 24k$. So, $x = 9k$. Therefore, the ratio $\frac{x}{S}$ is $\frac{9k}{15k} = \frac{3}{5}$ **<u>Ans.</u>**

23. *Given:* a rectangle composed of 4 squares. The area of the rectangle is 240.

Find: the perimeter.



Let x = the side length of the smallest square. The side length of the medium square is $2x$, and the side length of the largest square is $3x$. The rectangle has length $x + x + 3x = 5x$ and width $3x$. The area is $(5x)(3x) = 240$. Solving for x, we get $15x^2 = 240$, so $x^2 = 16$ and $x = 4$ (x can't be -4). Thus, the rectangle has length $5(4) = 20$, width $3(4) = 12$. The perimeter, then, is $2(20 + 12) = 64$ **<u>Ans.</u>**

24. *Find:* the least positive integer n such that $n!$ is divisible by 1000.

Since $1000 = 2^3 \times 5^3$, we are looking for n such that $n! = 2 \times 2 \times 2 \times 5 \times 5 \times 5 \times k$. With factors of 2 and 4 we have three 2s. With the factors of 5, 10 and 15, we have three 5s. The least value is $n! = 1 \times 2 \times 3 \times \cdots \times 14 \times 15$. So, $n = 15$ **<u>Ans.</u>**

25. *Given:* $\triangle$ABC is an isosceles triangle. AB = AC, $m\angle$A = 32°. Triangles ABC and PQR are congruent, and $m\angle$PXC = 114°.

Find: the degree measure of $\angle$PYC.

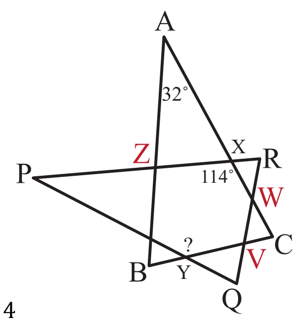


Figure 4

Since isosceles triangles ABC and PQR are congruent, AB = AC = PQ = PR, $m\angle P = m\angle A = 32°$ and $m\angle B = m\angle C = m\angle Q = m\angle R = \frac{180-32}{2} = \frac{148}{2} = 74°$. Since $m\angle$PXC = 114°, it follows that $m\angle$AXZ = 66°.

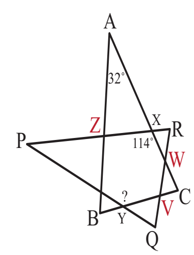


Figure 5

Now let's look at $\triangle$RXW and $\triangle$WCV, shown in Figure 5. By properties of vertical angles, $m\angle$RXW = $m\angle$AXZ. We also know that $m\angle RWX = 180 - (74 + 66) = 180 - 140 = 40°$. Again, by properties of vertical angles, $m\angle$VWC = $m\angle$RWX = 40°.

Since $m\angle RWX = 40°$ and $m\angle C = 74°$, it follows that $\triangle RXW \sim \triangle WCV$ (Angle-Angle) and $m\angle CVW = 66°$.

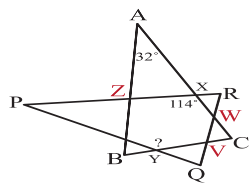


Figure 6

Now let's look at and $\triangle YVQ$, shown in Figure 6. Once again, by properties of vertical angles $m\angle YVQ = m\angle CVW = 66°$. Since $m\angle YVQ = 66°$ and $m\angle Q = 74°$, it follows that $\triangle WCV \sim \triangle YVQ$ and $m\angle VYQ = 40°$. Since angles PYC and VYQ are supplementary, we know that $m\angle PYC = 180 - 40 = 140°$ **<u>Ans.</u>**

26. *Given:* The integers 1-66 are arranged as shown.
Find: sum of the numbers in column D.

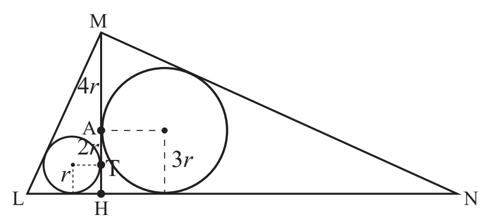
A	B	C	D	E
1	2	3		
		6	5	4
7	8	9		
		12	11	10
⋮	⋮	⋮	⋮	⋮

The numbers in column D all differ by 6 and range from 5 to 65. The sum of these numbers is $5 + 11 + \cdots + 59 + 65 =$ $70 \times \frac{11}{2} = 35 \times 11 = 385$ **<u>Ans.</u>**

27. *Given:* On Monday, a single worker painted a fence alone for two hours. Then two more painters came and, together, they finished the job 1.5 hours later. On Tuesday, a single worker began painting an identical fence at 8:00 a.m. Later 2 more workers showed up and, together, the 3 workers finished the fence at 10:54.
Find: the number of minutes the first worker painted alone on Tuesday.

All three workers paint at the same rate; we will call this R units of fences per hour. For Monday, when the entire job was completed in 3.5 hours, we have
$R \times 2 + 3R \times 1.5 = 6.5R$.
Now let T be the time worker 1 painted alone. For Tuesday, when the entire job was completed in 2.9 hours, we have
$R \times T + 3R \times (2.9 - T) = 8.7R - 2RT$.
Setting these two equations equal to each other and solving for T, we get
$6.5R = 8.74R - 2RT$ and $2.2R = 2RT$ so
$T = 1.1$ hours. Converting to minutes,
$1.1 \times 60 = 66$ **<u>Ans.</u>**

28. *Given:* $\triangle LMN$ has altitude MH. Circles are inscribed in triangles MNH and MLH, tangent to altitude MH.
MA:AT:TH = 4:2:1
Find: the ratio of the smaller circle's area to the larger circle's area.



Let r be the radius of the smaller circle. So TH = r, AT = $2r$ and MA = $4r$. It follows, then, that the radius of the larger circle has length AH = $2r + r = 3r$. The areas of the two circles are πx^2 and $\pi(3x)^2 = 9\pi x^2$, and the ratio of the areas is
$\frac{\pi x^2}{9\pi x^2} = \frac{1}{9}$ **<u>Ans.</u>**

29. *Given:* A bug crawls n miles at $n + 1$ mi/h on one day. The next day it crawls $2n + 1$ miles at $n^2 + n$ mi/h. The total time for the trip is 6 hours.
Find: the bug's average speed.
On the first day the bug spends $\frac{n}{n+1}$ hours crawling. On the second day, the bug spends $\frac{2n+1}{n^2+n}$ hours crawling. Since the entire trip took a total of 6 hours over the two days, we have $\frac{n}{n+1} + \frac{2n+1}{n^2+n} = 6$.
Solving for n, we get

$$\frac{n}{n+1} + \frac{2n+1}{n(n+1)} = 6$$

$$n + \frac{2n+1}{n} = 6(n+1)$$

$$n^2 + 2n + 1 = 6n^2 + 6n$$

$$5n^2 + 4n - 1 = 0$$

$$(5n-1)(n+1) = 0$$

$n = \frac{1}{5}$ (n can't be -1)

Substituting, we find that, on the first day, the bug crawled $\frac{1}{5}$ mile at $n + 1 = 1\frac{1}{5}$ mi/h. On the second day, it crawled $2n + 1 = 1\frac{2}{5}$ miles at $n^2 + n = \frac{1}{25} + \frac{1}{5} = \frac{6}{25}$ mi/h. This is a total of $\frac{1}{5} + 1\frac{2}{5} = 1\frac{3}{5}$ miles in 6 hours. So the average speed was $1\frac{3}{5} \div 6 = \frac{8}{5} \times \frac{1}{6} = \frac{8}{30} = \frac{4}{15}$ **Ans.**

30. *Find:* the greatest 5-digit palindrome n such that 7n is a 6-digit palindrome.
Start by looking at a 5-digit palindrome with leading digit 9 of the form 9__ __ __9.
Since 9 × 7 = 63, the last digit in the 6-digit product will be a 3, but there in no way to have the leading digit be a 3.
Next we check those with leading digit 8 of the form 8__ __ __8.
Since 8 × 7 = 56, the last digit in the 6-digit product will be a 6, and the leading digit could possibly be a 6 depending on what the other digits are.
The greatest palindrome we can form is 89998, but 7 × 89998 = 629986, which is not a palindrome. We continue looking at palindromes of the form 89__98 in descending order. Notice that with each successive palindrome of less value, the resulting 6-digit product of 7 and the palindrome decreases by 700. Also, the second digit in the product will always remain a 2 and the fifth digit will always be an 8. Next, looking at 88__88, the greatest palindrome we can form is 88988, but 7 × 88988 = 622916, which is not a palindrome. The fifth digit will always be a 1, so we are looking to bring the second digit down to a 1. We are subtracting 700 each time. We look at the second, third and fourth digit to figure out what multiple of 3 makes the second digit a 1 and the third and fourth digit equal.
229 − 63 = 166. We can get to the 6-digit palindrome 616616 by multiplying 7 and 88088 **Ans.**

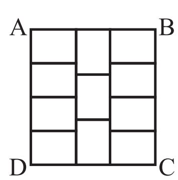
Target Round

1. *Given:* A season ski pass costs \$395. A day ticket costs \$40.
   Find: How much money is saved with a season ski pass compared to 38 day tickets?
   38 day tickets cost \$40 × 38 = \$1520
   \$1520 – \$395 = \$1125 **Ans.**

2. *Given:* A string of numbers consists of one 1, two 2s, three 3s, etc.
   Find: the 50th digit.
   The sum of 1 through 9 is 45. Therefore, starting with the 46th digit we have 10101.
   The 50th digit is 1 **Ans.**

3. *Given:* Eunice takes 2 minutes 30 seconds to run around a quarter-mile track.
   Find: How many minutes it takes her to run 1 mile.
   1 mile is 4 quarter-mile laps with each lap taking 2 minutes 30 seconds.
   2 laps take 5 minutes.
   4 laps take 10 minutes **Ans.**

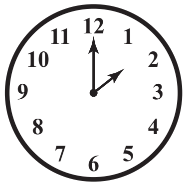
4. *Given:* Rectangle ABCD is composed of 11 congruent rectangles. The length of AB is 33.
   Find: the area of rectangle ABCD.



   Let x = the width of one of the rectangles.
   Let y = the length of the rectangle.
   IF AB has length 33,
   $2y + x = 33$
   The length of EF is the same as the length of AD. Therefore, $4x = 3y$ and $x = \frac{3}{4}y$
   Substituting back into the first equation:
   $2y + \frac{3}{4}y = 33$
   $\frac{11}{4}y = 33$
   $11y = 132$
   $y = 12$
   $EF = AD = 3y = 36$
   $36 \times 33 = 1188$ units^2 **Ans.**

5. *Given:* Sum of the first 5 terms of a sequence is 90 less than the sum of the next 5 terms.
   Find: the absolute difference between two consecutive terms of this sequence.
   Terms of an arithmetic sequence differ by the same amount.
   Let a = the first term and d be the common difference between consecutive terms. The sum of the first 5 terms is $a + a + d + a + 2d + a + 3d + a + 4d = 5a + 10d$. The sum of the next 5 terms is $a + 5d + a + 6d + a + 7d + a + 8d + a + 9d = 5a + 35d$. The difference between these two values is $(5a + 35d) – (5a + 10d) = 90$. Simplifying and solving for d yields $25d = 90$ and $d = \frac{90}{25} = \frac{18}{5}$ **Ans.**

6. *Given:* the hour and minute hands of a clock create a 60° angle at 2:00.
   Find: how many seconds later is the next time when the hour and minute hands create a 60° angle.
   The minute hand takes 1 hour, 3600 seconds, to go around the clock.
   Therefore, the minute hand moves $\frac{360}{3600} = \frac{1}{10}$° every second. The hour hand takes 12 hours, 12 × 3600 = 43200 seconds, to go around the clock. Therefore, the hour hand moves $\frac{6}{720} = \frac{1}{120}$° every second.
   Now let's look at a clock showing 2:00.



The minute hand is at $0°$ and the hour hand is at $60°$. Every second the minute hand moves $\frac{1}{10}°$ and the hour hand moves $\frac{1}{120}°$. This means that each second the minute hand comes $\frac{1}{10} - \frac{1}{120} = \frac{12}{120} - \frac{1}{120} = \frac{11}{120}°$ closer to the hour hand.
Let x = the number of seconds it will take for the two hands to coincide. Then $60 = \frac{11}{120}x$, and $x = \frac{120 \times 60}{11} = 654.\overline{54}$ seconds.
At this point the hands are exactly on top of each other. It will take the same amount of time for the minute hand to go past the hour hand to form a 60°angle.
$654.\overline{54} \times 2 = 1309.\overline{09} \approx 1309$ **Ans.**

7. *Given:* A bag contains some number of blue and exactly 11 yellow marbles. When 5 blue marbles are added, the probability of randomly drawing a blue marble exceeds 70%.
Find: the least possible number of blue marbles in the bag originally.
Let b = the original number of blue marbles. We have the following:

$$\frac{b+5}{b+5+11} > \frac{7}{10}$$
$$\frac{b+5}{b+16} > \frac{7}{10}$$

Cross-multiplying, we get
$10b + 50 > 7b + 112$. Solving for b yields $3b > 62$, so $b > 20\frac{2}{3}$. Substituting, the least integer greater than $20\frac{2}{3}$, we get

$$\frac{21+5}{21+16} = \frac{26}{37} \approx 70.207\%$$

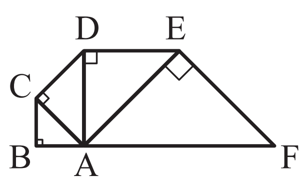
when b = 21 **Ans.**

8. *Given:* A room has eight light switches. Initially, exactly 5 of the lights are on. Three people enter the room sequentially and independently flip one switch randomly.
Find: the probability that after the third person has exited the room, exactly six of the lights are on.
In order to end up with exactly 6 lights on, the three people must turn two on and one off, in some order. Other options that would not result in six lights on are turning on three lights, turning off three lights or turning one on and two off.
There are 3 × 2 × 1 = 6 ways to turn three lights on. This is because when the first person enters the room the number of lights off is 3, when the second person enters the room there are 2 lights off and when the third person enters the room there is 1 light off.
Similarly, there are 5 × 4 × 3 = 60 ways to turn three lights off.
There are 5 × 4 × 5 = 100 ways to turn two lights off then one on, in that order.
There are 5 × 4 × 5 = 100 ways to turn one light off, one light on and one light off.
There are 3 × 6 × 5 = 90 ways to turn one light on then two lights off.
There are 3 × 2 × 7 = 42 ways to turn two lights on then one off.
There are 3 × 6 × 3 = 54 ways to turn one light on, one light off then one light on.
There are 5 × 4 × 3 = 60 ways to turn one light off then two lights on.
There are a total of 6 + 60 + 100 + 100 + 90 + 42 + 54 + 60 = 512 ways for three of the eight switches to be flipped.
Of these 512 ways, 42 + 54 + 60 = 156 ways to end with exactly 6 lights on.
The probability of exactly 6 lights being on after the third person exits the room is $\frac{156}{512} = \frac{39}{128}$ **Ans.**

Team Round

1. *Given:* A package of 8 frankfurters costs \$5. A package of 12 buns costs \$3.
   Find: the percent of the cost of a hot dog sandwich that comes from a bun.
   A frankfurter costs $\frac{5}{8} = \$0.625$
   A bun costs $\frac{3}{12} = \$0.25$
   $\frac{0.25}{0.625 + 0.25} = \frac{0.25}{0.875} \approx 0.285714 =$ $28.5714\% \approx 29\%$ **Ans.**

2. *Given:* Triangles ABC, ACD, ADE and AEF are isosceles right triangles.
   Find: the value of $\frac{AB}{AF}$.



   Let *x* = the length of AB. AC is the hypotenuse of triangle ABC.
   Since all the triangles are 45-45-90, we know the ratio of the side to the hypotenuse is 1 to $\sqrt{2}$. So AC = $x\sqrt{2}$, AD = $2x$, AE = $2\sqrt{2}x$ and AF = $4x$.
   $\frac{AB}{AF} = \frac{x}{4x} = \frac{1}{4}$ **Ans.**

3. *Find:* the 6-digit number that has these properties:
   - None of the digits repeat.
   - The ones digit is a prime number.
   - The ones digit is the sum of the tens and hundreds digits.
   - The thousands digit is the sum of the hundreds and ten-thousands digits.
   - The hundred-thousands digit is neither prime nor composite.
   - The sum of the digits is 21.

   The hundred-thousands digit is not prime, so it cannot be 2, 3, 5 or 7. It is also not composite. So it can't be 4, 6, 8 or 9. Therefore, it must be 1.
   The ones digit is prime. Therefore, it can be 2, 3, 5, or 7. But it is also the sum of the tens and hundreds digit. So the ones digit cannot be 2 because that would make the tens and hundreds digit both be 1.
   The ones digit also cannot be 3 because then one of the tens or hundreds digits would be 1 and that is already taken by the hundred thousands digit.
   So the ones digit can be 5 or 7.
   Suppose the ones digit is 5.
   Then the sum of the tens and hundreds digits is 5. That could be 1 and 4 or 2 and 3. 1 is already used. So let's now suppose that the tens digit is 3 and the hundreds digit is 2.
   The sum of all the digits is 21. We have already used 1, 2, 3, and 5. So the other two values must add up to 10. That could be 1 and 9, 2 and 8, 3 and 7, or 4 and 6. Only 4 and 6 haven't been used.
   If the thousands digit is 6, it would have to be the sum of the hundreds digit (2) and the ten-thousands digit, which would have to be 4. The number: 146,235 **Ans.**

4. *Given:* a bicycle tire has a diameter of 22 inches. A motorcycle tire has a diameter of 25 inches.
   Find: how many more feet the motorcycle travels than the bicycle after each tire has made 1000 revolutions.
   After 1000 revolutions of the tire, the bicycle traveled 22,000π inches. The motorcycle traveled 25,000π inches. That's a difference of 3000π inches.
   Converting to feet: $\frac{3000\pi}{12} = 250\pi$ **Ans.**

5. *Find:* the sum of the positive integers from 1 through 500 that are divisible by 2, 3, 4, 5 and 6.

The LCM of 2, 3, 4, 5 and 6 is $2 \times 2 \times 3 \times 5 = 60$. The multiples of 60 between 1 and 500 are 60, 120, 180, 240, 300, 360, 420 and 480. There sum is $60 + 120 + 180 + 240 + 300 + 360 + 420 + 480 =$
$540 \times 4 = 2160$ **<u>Ans.</u>**

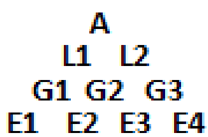
6. *Given:* the mean, median, unique mode and range of 10 integers are all 10.
   Find: greatest possible integer in the list.
   Since the mean is 10, the sum of the 10 integers is 100. The median is 10 which means that the fifth and sixth integers are both 10 or the average of the fifth and sixth integers is 10. However, since the mode is also 10, the fifth and sixth integers must be 10. This means the largest the minimum value could possibly be would be 10, and since the range is 10, this would make the maximum 20. The list of integers would be 10, 10, 10, 10, 10, 10, 10, 10, 10, 20, but this doesn't work since this sums to 110. So how about 19 as the maximum?
   We can start with 9 and end with 19.
   $9 + 19 = 28$
   That leaves 8 integers between 10 and 18 to sum to $100 - 28 = 72$
   That's not going to work.
   How about a high of 18?
   That will work.
   We can start with 8 and end with 18.
   $8 + 18 = 26$
   That leaves 8 integers to sum to
   $100 - 26 = 74$
   8, 8, 8, 8, 10, 10, 10, 10, 10, 18
   These 10 numbers sum to 100 for a mean of 10. The median is 10, the mode is 10 and the range is 10. The greatest possible integer is 18 **<u>Ans.</u>**

7. *Given:* A 200 milliliter solution is 7% detergent.
   Find: How many milliliters of 100% detergent need to be added so the solution will be 14% detergent?
   7% of 200 milliliters is 14 milliliters of detergent.
   Let x = the amount we need to add to get a solution of 14% detergent.
   $$\frac{14 + x}{200 + x} = 0.14$$
   $14 + x = 28 + 0.14x$
   $0.86x = 14$
   $x = \frac{14}{0.86} \approx 16.27906 \approx 16.3$ ml **<u>Ans.</u>**

8. *Find:* how many 3-digit numbers are there such that each of the digits is prime and the sum of the digits is prime?
   The prime single digit numbers are 2, 3, 5 and 7.
   Consider, first, that each digit is different. Forget using any other combination with one 2 because two odds plus one even will always be an even number.
   $3 + 5 + 7 = 15$
   This isn't prime.
   Now consider that two of the digits are the same.
   $2 + 2 + 3 = 7$
   That works and there are 3 permutations.
   $2 + 2 + 5 = 9$
   That doesn't work.
   $2 + 2 + 7 = 11$
   That works. There are 3 permutations.
   $3 + 3 + 5 = 11$
   That works. There are 3 permutations.
   $3 + 3 + 7 = 13$
   That works. There are 3 permutations.
   $5 + 5 + 3 = 13$
   That works. There are 3 permutations.
   $5 + 5 + 7 = 17$
   That works. There are 3 permutations.
   $7 + 7 + 3 = 17$

That works. There are 3 permutations.
$7 + 7 + 5 = 19$
That works. There are 3 permutations.
We do not need to consider 3 of the same prime because the sum of 3 of the same prime is always going to be divisible by 3.
So we now have $8 \times 3 = 24$ numbers **Ans.**

9. *Find:* How many different paths from top to bottom spell ALGEBRA?



From the first line (with A) to the second line (with L) there are two paths (e.g., 2^1): AL1 and AL2.
From the second line to the third line (with G) there are 4 paths:
AL1G1, AL1G2, AL2G2, AL2G3 (e.g., 2^2).
And we can see the pattern.
There are 2^3 paths to the fourth line (with E), 2^4 paths to the fifth line (with B), 2^5 paths to the sixth line (with R) and 2^6 paths to the seventh line (with A).
$2^6 = 64$ **Ans.**

10. *Given:* The number of tokens Devin has is a square number less than 100. If Kevin gives Devin his tokens, Devin will have a total number of tokens that is still a square. If Devin gives Kevin the same number of tokens that Kevin already has, the number of tokens Devin is left with is also square.
Find: How many tokens does Kevin have?
List the number of squares less than 100:
1, 4, 9, 16, 25, 36, 49, 64, 81
For Devin to give Kevin the same number of tokens that Kevin already has, Devin must have more tokens than Kevin.
So what we have is the following:
Let d = the number of tokens Devin has
Let k = the number of tokens Kevin has
$d + k$ is a square
$d - k$ is a square
$d > k$
d cannot be 1 or 4, since Kevin would have more tokens than Devin.
Suppose $d = 9$
$9 + 7 = 16; 9 - 7 = 2$
This cannot work. There are no more values for k where $d > k$.
Suppose $d = 16$
$16 + 9 = 25; 16 - 9 = 7$
This cannot work. There are no more values for k where $d > k$.
Suppose $d = 25$.
$25 + 11 = 36; 25 - 11 = 14$
$25 + 24 = 49; 25 - 24 = 1$
And we have it.
$k = 24$ **Ans.**