

MATHCOUNTS®

2018

■ Chapter Competition ■ Sprint Round Problems 1–30

HONOR PLEDGE

I pledge to uphold the highest principles of honesty and integrity as a Mathlete®. I will neither give nor accept unauthorized assistance of any kind. I will not copy another's work and submit it as my own. I understand that any competitor found to be in violation of this honor pledge is subject to disqualification.

Signature _____ Date _____

Printed Name _____

School _____

DO NOT BEGIN UNTIL YOU ARE INSTRUCTED TO DO SO.

This section of the competition consists of 30 problems. You will have 40 minutes to complete all the problems. You are not allowed to use calculators, books or other aids during this round. If you are wearing a calculator wrist watch, please give it to your proctor now. Calculations may be done on scratch paper. All answers must be complete, legible and simplified to lowest terms. Record only final answers in the blanks in the left-hand column of the competition booklet. If you complete the problems before time is called, use the remaining time to check your answers.

In each written round of the competition, the required unit for the answer is included in the answer blank. The plural form of the unit is always used, even if the answer appears to require the singular form of the unit. The unit provided in the answer blank is the only form of the answer that will be accepted.

Total Correct	Scorer's Initials

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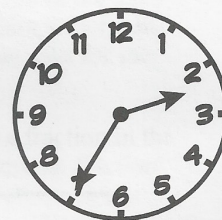
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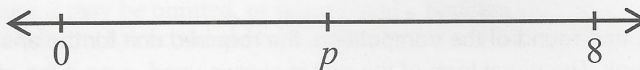
1. _____ What integer is closest in value to 2017.7012?

2. _____ minutes How many minutes past 2 o'clock is the time displayed on the clock shown here?



3. \$ Rufus has \$835 in his checking account. After he pays \$415 for his portion of the rent and \$220 for his portion of the utilities, how much does Rufus have left in his checking account?

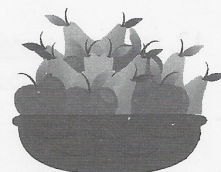
4. _____ On the number line shown, p is halfway between 0 and 8. What is the value of p ?



5. \$ What is the total cost of 2 large bowls priced \$14 each and 14 small jars priced \$2 each?

6. _____ The sum of an integer n and four is equal to the difference of ten and two. What is the value of n ?

7. _____ apples The ratio of apples to pears in Celia's basket is 3:5. If there are 20 pears, how many apples are there in Celia's basket?



8. _____ If $462 = 7 \times 11 \times p$, what is the value of p ?

9. _____ What is the value of $\frac{1}{0.25} - 2$?

10. _____ The arithmetic mean of 9, 11, 13, x and 7 is 10. What is the value of x ?

11. _____ If p and q are real numbers and $3(p + 4) + 4(p + 11) = 7(p + q)$, what is the value of q ?
12. _____ in³ A box of Yummy Number Crunch is a rectangular prism with height 10 inches, width 8 inches and depth 3 inches. What is the volume of the box?
13. _____ If $\frac{z}{16} = \frac{4}{z}$, what is the positive integer value of z ?
14. _____ cents Meredith buys a bag containing eight apples for \$3.50. When she gets home, she realizes one of the apples is rotten, so she throws it away. What is the actual cost for each non-rotten apple, in cents?
15. _____ By switching two of the digits of the number 123,456, Rodrigo obtains a new number that is 1980 more than the original. What is the product of the two digits Rodrigo switched?

16. _____ degrees In triangle ABC, $m\angle A = 35$ degrees and $m\angle B = 95$ degrees. What is the measure of $\angle C$?

17. _____ amounts Dak has a quarter, a dime, a nickel and a penny. How many different amounts can be obtained by using one or more of the coins in Dak's collection?



18. _____ The sum of two numbers is 14, and their difference is 4. What is their product?

19. _____ What is the value of $16^2 - 15^2$?

20. _____ The arithmetic mean of a and b is 6.2, the mean of b and c is 7.3 and the mean of a and c is 4.5. What is the value of $a + b + c$?

21. _____ What is the greatest integer value of x for which $3^x < 100$?

22. _____ Which digit should replace A in the units place of the number 987,654,32A so that the number is divisible by 6?

23. _____ rectangles How many rectangles of any size are in the figure shown?



24. _____ In the equation shown, if p and q are positive integers and p is odd, what is the value of the sum $p + q$?

$$\frac{1}{2} \cdot \frac{3}{4} \cdot \frac{5}{6} \cdot \frac{7}{8} \cdot \frac{9}{10} \cdot \frac{11}{12} \cdot \frac{13}{14} \cdot \frac{15}{16} \cdot \frac{17}{18} \cdot \frac{19}{20} = \frac{p}{2^q}$$

25. \$ _____ Three employees split a bonus valued at some number of dollars. Arman first receives \$10 more than one third of the total amount. Bernardo then receives \$3 more than one half of what was left. Carson receives the remaining \$25. What is the total dollar value of the bonus?

26. _____ A line is tangent to a circle at the point (2, 5). A parallel line is tangent to the circle at (8, 13). What is the common slope of the two lines? Express your answer as a common fraction.
27. _____ For a particular list of four distinct integers the mean, median and range have the same value. If the least integer in the list is 10, what is the greatest value for an integer in the list?
28. _____ units^2 In circle P, chords AB and CD intersect at E so that $AD = AE = 3$, $ED = 4$ and $BC = 5$. What is the area of triangle BCE? Express your answer as a common fraction in simplest radical form.
29. _____ There are two values of x such that $\frac{|x-2018|}{|x-2019|} = \frac{1}{6}$. What is the absolute difference between these two values of x ? Express your answer as a common fraction.
30. _____ Six standard six-sided dice are rolled, and the sum S is calculated. What is the probability that $S \times (42 - S) < 297$? Express your answer as a common fraction.

Forms of Answers

The following list explains acceptable forms for answers. Coaches should ensure that Mathletes are familiar with these rules prior to participating at any level of competition. Judges will score competition answers in compliance with these rules for forms of answers.

Units of measurement are not required in answers, but they must be correct if given. When a problem asks for an answer expressed in a specific unit of measure or when a unit of measure is provided in the answer blank, equivalent answers expressed in other units are not acceptable. For example, if a problem asks for the number of ounces and 36 oz is the correct answer, 2 lbs 4 oz will not be accepted. If a problem asks for the number of cents and 25 cents is the correct answer, \$0.25 will not be accepted.

All answers must be expressed in simplest form. A "common fraction" is to be considered a fraction in the form $\pm \frac{a}{b}$, where a and b are natural numbers and $\text{GCF}(a, b) = 1$. In some cases the term "common fraction" is to be considered a fraction in the form $\frac{A}{B}$, where A and B are algebraic expressions and A and B do not share a common factor. A simplified "mixed number" ("mixed numeral," "mixed fraction") is to be considered a fraction in the form $\pm N\frac{a}{b}$, where N , a and b are natural numbers, $a < b$ and $\text{GCF}(a, b) = 1$. Examples:

Problem: What is $8 \div 12$ expressed as a common fraction? *Answer:* $\frac{2}{3}$ *Unacceptable:* $\frac{4}{6}$
Problem: What is $12 \div 8$ expressed as a common fraction? *Answer:* $\frac{3}{2}$ *Unacceptable:* $\frac{12}{8}$, $1\frac{1}{2}$
Problem: What is the sum of the lengths of the radius and the circumference of a circle with diameter $\frac{1}{4}$ unit expressed as a common fraction in terms of π ? *Answer:* $\frac{1+2\pi}{8}$
Problem: What is $20 \div 12$ expressed as a mixed number? *Answer:* $1\frac{2}{3}$ *Unacceptable:* $1\frac{8}{12}$, $\frac{5}{3}$

Ratios should be expressed as simplified common fractions unless otherwise specified. Examples:

Simplified, Acceptable Forms: $\frac{7}{2}$, $\frac{3}{\pi}$, $\frac{4-\pi}{6}$ *Unacceptable:* $3\frac{1}{2}$, $\frac{1}{3}$, 3.5, 2:1

Radicals must be simplified. A simplified radical must satisfy: 1) no radicands have a factor which possesses the root indicated by the index; 2) no radicands contain fractions; and 3) no radicals appear in the denominator of a fraction. Numbers with fractional exponents are *not* in radical form. Examples:

Problem: What is the value of $\sqrt{15} \times \sqrt{5}$? *Answer:* $5\sqrt{3}$ *Unacceptable:* $\sqrt{75}$

Answers to problems asking for a response in the form of a dollar amount or an unspecified monetary unit (e.g., "How many dollars...," "How much will it cost...," "What is the amount of interest...") should be expressed in the form (\$) $a.bc$, where a is an integer and b and c are digits. The *only* exceptions to this rule are when a is zero, in which case it may be omitted, or when b and c both are zero, in which case they both may be omitted. Answers in the form (\$) $a.bc$ should be rounded to the nearest cent unless otherwise specified. Examples:

Acceptable: 2.35, 0.38, .38, 5.00, 5 *Unacceptable:* 4.9, 8.0

Do not make approximations for numbers (e.g., π , $\frac{2}{3}$, $5\sqrt{3}$) in the data given or in solutions unless the problem says to do so.

Do not perform any intermediate rounding (other than the "rounding" a calculator does) when calculating solutions. All rounding should be done at the end of the computation process.

Scientific notation should be expressed in the form $a \times 10^n$ where a is a decimal, $1 \leq |a| < 10$, and n is an integer. Examples:

Problem: What is 6895 expressed in scientific notation? *Answer:* 6.895×10^3
Problem: What is 40,000 expressed in scientific notation? *Answer:* 4×10^4 or 4.0×10^4

An answer expressed to a greater or lesser degree of accuracy than called for in the problem will not be accepted. Whole number answers should be expressed in their whole number form.

Thus, 25.0 will not be accepted for 25, and 25 will not be accepted for 25.0.

The plural form of the units will always be provided in the answer blank, even if the answer appears to require the singular form of the units.