Girls in Math

Name: ____________________ **Team:** ____________________

2025 Individual Round

ID: __________

1. ________
2. ________
3. ________
4. ________
5. ________
6. ________
7. ________
8. ________
9. ________
10. ________
11. ________
12. ________

– 75 minutes
– no calculators
– positive integer answers

1. Juno has a square sheet of paper. She folds the paper along the diagonal to form an isosceles right triangle. Then she folds the triangle in half along the median to its hypotenuse 3 more times. After this is done, the hypotenuse of the right triangle has length 5. What is the area of the original square?

2. The nth *primorial* is the product of the first n prime numbers. For example, the first *primorial* is 2, and the second *primorial* is 6. The nth *priprimorial* is the product of the first n *primorials*. How many divisors does the fifth *priprimorial* have?

3. Eight people are standing in a line and each person is labeled according to their position in the line, from person 1 to person 8. Each person is friends with some of the other people in the line. Person n has n friends if n is odd and $\frac{n}{2}$ friends if n is even. Friendships are mutual. What is the sum of the numbers of person 3's friends?

4. Penelope the Penguin lives in a regular hexagonal barn with a side length of 7 meters, where she keeps her pet seal, Sarah. Sarah has a 15-meter long leash tied on the outside one of the barn's corners and cannot enter the barn. The total area in which Sarah can roam, given the length of the leash and the shape of the barn, can be expressed as $\frac{a\pi}{b}$ for relatively prime positive integers a, b. What is $a + b$?

5. Define a function f by $f(1) = 0$ and $f(np) = f(n) + p$ for primes p, and positive integers n. What is the value of $f(2025^{2025})$?

6. Elizabeth and Elise are playing a game. They take turns rolling a fair 6-sided die. The round ends when one player rolls a multiple of 3. If the round ends on a player's nth turn, that player gets n points and the other gets 0. They play twice and each player goes first once. The expected number of points that Elise has at the end can be expressed as $\frac{m}{n}$ for m, n relatively prime positive integers. What is $m + n$?

7. The sum
$$\sum_{k=2}^{100} \log\left(1 - \frac{1}{k^2}\right)$$
can be written as $\log\left(\frac{a}{b}\right)$, where a and b are positive, relatively prime integers. Find $a + b$.

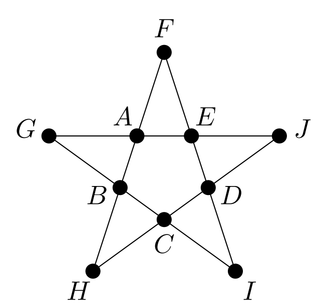
8. Define $n!!$ to be the product of all positive integers less than n with the same parity as n. More formally, $n!! = n(n-2)(n-4)\ldots(n-2k)$, where k is the largest integer such that $n - 2k > 0$. Let $a_1, a_2, \ldots a_{100}$ be a sequence of monic polynomials where for all n, $\deg(a_n(x)) = n$, $a_n(n) = (n!!)^2$, and $a_{n-1}(x)$ is a factor of $a_n(x)$. If each function $a_n(x)$ intersects the x-axis c_n number of times, what is $c_1 + c_2 + \ldots + c_{100}$?

9. Alice generates 12 real numbers uniformly at random from the interval $[0, 1]$. Bob generates 13 real numbers uniformly at random from the interval $[0, 1]$ and then deletes the lowest number. The next day, they realized that they didn't put their names on their datasets, so they forget who generated which one. Bob guesses that the dataset whose minimum is larger is his. Let $\frac{a}{b}$ be the probability that he's right for relatively prime positive integers a and b. What is $a + b$?

10. Let $f(x) = x^4 - 2x^3 + x^2 - 8x + 128$, and let y, a, l, and e be real numbers such that $y + a + l + e = 3$. What is the minimum value of $f(y) + f(a) + f(l) + f(e)$?

11. Let ABC be a triangle with $AB = 20$ and $AC = 25$. Let D be the midpoint of AB, E be the midpoint of AC, and F be any point on BC. The circumcircles of $\triangle BFE$ and $\triangle CFD$ intersect at a point $P \neq F$. It is known that $\angle DPE = 90°$. If d is length of BC, find d^2.

12. Katherine is strolling a star-shaped grid (shown in diagram). Katherine begins at vertex A, and at every step, Katherine randomly moves to any of the vertices immediately adjacent to her current vertex, *except* for the vertex she just came from on the previous step. For example, on step 1, Katherine has equal probability of moving to any of B, E, F, G. Suppose Katherine moved to B; then on step 2, she has equal probability of moving to any of C, H, G.



What is the probability that Katherine travels in a 5-cycle (for example, $A \to E \to D \to C \to B \to A$) before she travels in any 3-cycle (for example, $A \to F \to E \to A$)?