Girls in Math

Name: ______________________ **Team:** ______________________

2025 Team Round

ID: __________

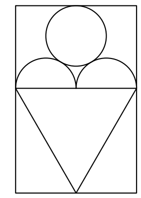
1. __________
2. __________
3. __________
4. __________
5. __________
6. __________
7. __________
8. __________
9. __________
10. __________
11. __________
12. __________

– 45 minutes
– no calculators
– positive integer answers

1. Cat and Claire are discussing Cat's favorite number. Cat says, ”My favorite number is a two-digit prime.” Claire asks, ”If you told me both digits of the number but not their order, would I know your number?” Cat says, ”No. But if I had told you both digits as well as the fact that my number is also one less than twice a perfect square, I know that you still will not know my number.”
Find the sum of the possible numbers that are Cat's favorite number.

2. A penguin has the expression $5?5?5?5?5$. If the penguin replaces each ”?” with ”+,” ”−,” or ”÷,” what is the smallest possible nonnegative integer value the expression can have (assuming standard order of operations; no additional parenthesis can be added)?

3. Isla paints an ice cream cone on a blank canvas as shown on the right. The scoops are two half circles and one circle, each of radius 4 inches. The cone is an equilateral triangle. The area of the canvas that is left unpainted can be represented in the form $a + b\sqrt{c} + d\pi$ where $a, b, c,$ and d are all integers and c is not divisible by any perfect square. Find $a + b + c + d$.



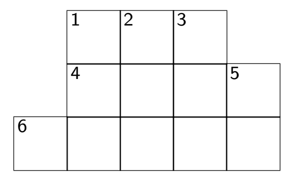
4. Sally's Sweets is running a special weekly promotion, where customers can trade in 3 Sally's chocolate wrappers for another Sally's chocolate. If Irene wants to eat 100 chocolates in total, what's the minimum amount of chocolates she must buy initially at Sally's Sweets?

5. Tic-Tac-Toe 3D is played on a 3x3x3 grid. Two players alternate, putting marks in one cell in the grid at a time. Players try to win (if they cannot win, they aim to tie, and if they are guaranteed to lose, they delay the game for as long as possible. If they are guaranteed to win, they try to win as fast as possible). If both players play optimally, after how many moves is the game guaranteed to end?

6. See the following math crossword with the following clues:

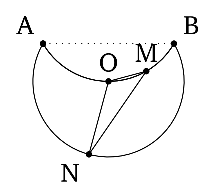
Across
- 1. Is a perfect square
- 4. Contains one digit that is equal to the sum of its remaining digits
- 6. Is divisible by 7, 11, and 13

Down
- 1. Is divisible by 23
- 2. Is a Fibonacci number
- 3. Is divisible by 17
- 5. Is a prime number
- 6. Is the sum of the digits of the binary representation of 2025



If there are no leading zeroes, what is the sum of the answers to all eight clues?

7. Yushu likes origami. She has a paper circle of radius 4, and folds the outer edge onto the center O (see diagram). Let M be the midpoint of arc OB, and $\angle AON = 105°$. The area of $\triangle MON$ can be expressed as $a\sqrt{b} + c\sqrt{d}$ for integers a, b, c, d, with b, d containing no square factors. What is $a + b + c + d$?



8. Alice and Bob plays a game with 1000 cards in a row on a table, face-down. One of the cards is marked and the others are blank, and Alice knows which card is the marked one. Bob will then pick m cards. If any of these are the marked card, he wins. Otherwise, he gets to choose a set of n cards, and shuffle them *randomly*. Then, Alice picks one card. If it is the marked card, Alice wins, otherwise, Bob wins.
Given that Alice picks optimally and Alice and Bob are equally likely to win, what is $m + n$?

9. Call a set S of 3 digits (integers from 0 to 9, inclusive) *representative* if for all integers $n \geq 1$, at least one digit in S appears in the base 10 representation of n or $3n$. There exists exactly two distinct *representative* sets, say $\{a_1, a_2, a_3\}$ and $\{b1, b2, b3\}$. Compute $a_1a_2 + a_2a_3 + a_3a_1 + b_1b_2 + b_2b_3 + b_3b_1$.

10. Lily is rolling a shrinking die! Every step, when Lily rolls a number k, her die for the subsequent step becomes k-sided. She begins by rolling a 25-sided die on step 0, and repeats this process until she eventually holds a 1-sided die. Let $\{k_0, k_1, \ldots k_l\}$ denote the set of *unique* dice sizes she has at some point, with $k_0 = n, k_l = 1$. Find $\mathbb{E}[\sum_{i=0}^{l} k_i^2]$.

11. Define an infinite sequence $\{a_n\}$ by $a_1 = 1$ and $a_n = 3^{a_{n-1}} - 1$ for all $n \geq 2$. Let $\tau(n)$ count the number of divisors of n. Find the number of terms a_i that satisfy $3a_i \geq \tau(a_{i+1})$.

12. Consider triangle ABC with $AB = 5, BC = 7, AC = 8$. Denote the incircle of $\triangle ABC$ as ω and let ω meet AB, AC, BC at D, E, F respectively. Draw circle γ such that γ passes through A and tangents to BC at F. Denote the intersections of line DE and γ as X and Y, the length of XY can be written in the simplest form of $\frac{p\sqrt{q}}{r}$, compute $p + q + r$.