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MATHCOUNTS®

2026 State Competition

Sprint Round Problems 1–30

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HONOR PLEDGE

I pledge to uphold the highest principles of honesty and integrity as a Mathlete®. I will neither give nor accept unauthorized assistance of any kind. I will not copy another's work and submit it as my own. I understand that any competitor found to be in violation of this honor pledge is subject to disqualification.

Signature ____ Date _______________

Printed Name ____

School ____

DO NOT BEGIN UNTIL YOU ARE INSTRUCTED TO DO SO.

This section of the competition consists of 30 problems. You will have 40 minutes to complete all the problems. You are not allowed to use calculators, books or other aids during this round. Calculations may be done on scratch paper. All answers must be complete, legible and simplified to lowest terms. Record only final answers in the blanks in the left-hand column of the competition booklet. If you complete the problems before time is called, use the remaining time to check your answers.

In each written round of the competition, the required unit for the answer is included in the answer blank. The plural form of the unit is always used, even if the answer appears to require the singular form of the unit. The unit provided in the answer blank is the only form of the answer that will be accepted.

Total Correct	Scorer's Initials

TITLE SPONSORS

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FOUNDING SPONSORS: National Society of Professional Engineers, National Council of Teachers of Mathematics and CNA Insurance

1. ____________ plots

Ms. Souza's fourth grade class is going to start a garden project. The field of land the class has to work with is 75 feet by 40 feet. They plan to break the field into 2 foot by 3 foot plots, with no plots overlapping. What is the maximum number of plots of these dimensions that they will be able to create out of the field?

2. ____________ clubs

Maxwell is signing up for three afterschool clubs on different school days of the week. If Maxwell is signing up for the *math* and *chess* clubs, then how many clubs can Maxwell select from as his third club?

Afterschool Clubs

Monday	Tuesday	Wednesday	Thursday	Friday
Robotics Debate Drama Chess	Coding Writing Reading History Astronomy Baseball	Spanish Math Photography Culinary Film Gardening Business	Journalism Fitness Music/Band	Olympiad Baking Poetry Fashion French

3. ____________

What is the sum of the three distinct prime numbers that are divisors of $29^2 - 17^2$?

4. ____________ candies

Sally ate 2 of her candies and gave half of the leftover candies to her friend. If she has 24 candies left, how many candies did she originally have?

5. ____________

If $x + y = 5$ and $x^2 - y^2 = 15$, what is the value of x?

6. ______________ The sum of two different positive integers is 8 more than the absolute difference between the two integers. What is the least possible value of the sum of the two integers?

7. ____________ mi Max is driving at a constant rate of 55 miles per hour. How many miles does Max travel in three minutes? Express your answer as a decimal to the nearest hundredth.

8. ____________ oz Yaretzy is baking cookies that have the same thickness. She notices that 10 ounces of dough will be exactly enough to make a round cookie that is 10 inches in diameter. How many ounces of dough does she need to make a round cookie that is 16 inches in diameter, with the same thickness as the 10-inch cookie? Express your answer as a decimal to the nearest tenth.

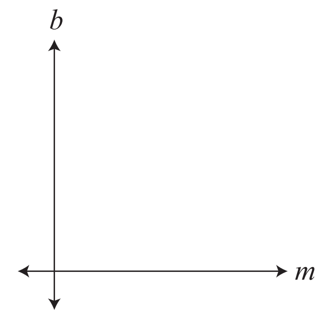
9. ______________ Jonathon lists out the first 10 Fibonacci numbers: 1, 1, 2, 3, 5, 8, 13, 21, 34, 55. What is the sum of the mean, median and mode of these numbers? Express your answer as a decimal to the nearest tenth.

10. ________ integers How many two-digit positive integers are 36 more than the sum of their digits?

11. ______________ What is the remainder when 23^{23} is divided by 5?

12. ______________ If $x + \frac{x}{2} + \frac{x}{2^2} + \frac{x}{2^3} = 30$, what is the value of x?

13. ______________ Using the equation $6 = 2m + b$, all ordered pairs (m, b) are plotted in the mb-plane, forming a line. What is the area of the triangle formed by this line, the line $m = 0$ and the line $b = 0$?



14. ______________ integers For how many positive integers n is there <u>no</u> positive integer strictly between one-half of n and three-fifths of n?

15. ______________ ft² Andrew sells Mia a rectangular plot of land with an area of 60 square feet and all sides measuring a whole number of feet. Mia extends the width of the plot by 4 feet, while shortening the length by 2 feet. What is the greatest possible area of Mia's new plot of land?

16. ______________ An isosceles triangle has two side lengths of 20 and 26. What is the sum of the squares of the possible lengths of the altitude to the side of the triangle whose length is different from those of the other two sides?

17. ______________ Fry and Amy are playing ping-pong. They take turns serving, starting with Fry. When Fry serves, there is a 50% chance he will score a point and a 50% chance Amy will score a point. When Amy serves, there is a 70% chance she will score a point and a 30% chance Fry will score a point. They keep playing until one of them scores 4 points. What is the probability Fry serves at most 3 times? Express your answer as a common fraction.

18. ______________ If $36^{x+1} = 6^8$, what is the value of x?

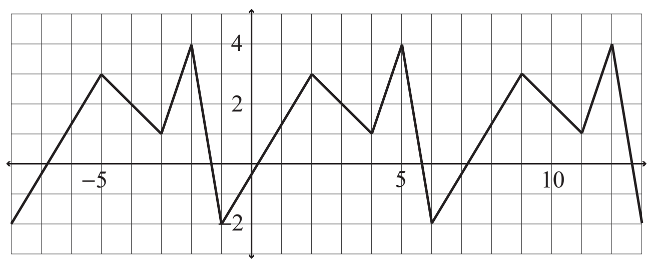
19. ______________ % Regular hexagon ABCDEF has side length 2. Let P, Q and R be the midpoints of segments AB, CD and EF, respectively. What percent of the area of ABCDEF is the area of PQR? Express your answer to the nearest whole percent.

20. ______________ arrangements How many distinct arrangements of the letters in the word PROBABILITY contain the sub-arrangement PRO, whose letters must appear consecutively and in that order anywhere in the arrangement?

21. ______________ Six students are seated in a single row of seats with an aisle at each end. The students leave their seats one at a time in random order. What is the probability that each student is able to move to one of the aisles without having to cross over another seated student? Express your answer as a common fraction.

22. ________ integers The four-digit integer 2018 is composed of four distinct digits whose sum is 11. Including 2018, how many positive four-digit integers are composed of four distinct digits whose sum is 11?

23. ________ inter-sections Let $f(x)$ be the periodic function shown, such that $f(x) = f(x + 7)$ for all real values of x and let $g(x) = f(x^2)$. How many intersections of the graphs of $y = g(x)$ and $y = 2$ occur in the interval $20 \le x \le 22$?

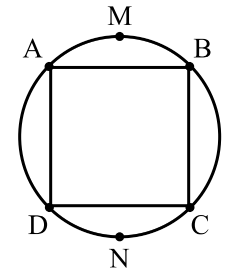


24. ______________ In a certain sequence, $a_0 = 20$ and $a_n = \sqrt{(a_{n-1})^2 + 1}$ for any positive integer n. What is the value of a_{500}?

25. ________ combi-nations Andrea has a deck of 50 cards consisting of ten cards, numbered 1 through 10, in each of five distinct colors. She randomly draws five cards from this deck and, without looking, shows all five cards to Oscar. Oscar then tells Andrea that exactly three of the five cards are blue, and exactly two of the five cards show the number 5. How many distinct unordered combinations of five cards drawn from the deck, consistent with Oscar's statements, do not contain the blue 5?

26. ____________ in^2

Square ABCD with side length 4 inches is inscribed in a circle. Let M and N be the midpoints of minor arcs AB and CD, respectively. What is the value of AM × AN? Express your answer in simplest radical form.



27. ____________

If x is a positive real number satisfying $x^x = 3$, what is $x^{x^{x+1}}$?

28. ____________

What is the sum of all prime factors of $2^{49} - 2$ that can be written in the form $2^n + 1$ for some integer n?

29. ____________

Vasudha picks up a stick of length 20 inches. She breaks it at a point exactly 3 inches from an endpoint and puts the smaller piece down. She then selects two random points on the larger piece and breaks the piece at those two points. What is the probability that the four pieces she creates can form the side lengths of a quadrilateral? Express your answer as a common fraction.

30. ____________

Ford has an unfair random number generator that generates positive integers. When the generator is run, each positive integer n comes up with probability $\dfrac{1}{n(n+1)}$. For example, the probability of generating an 8 is $\dfrac{1}{72}$. Ford calls a positive integer "interdimensional" if it is one less than a triangular number. (The first few interdimensional numbers are 2, 5, 9, 14, …) What is the probability that Ford's random number generator shows an interdimensional number when run? Express your answer as a common fraction.

Forms of Answers

The following list explains acceptable forms for answers. Coaches should ensure that Mathletes are familiar with these rules prior to participating at any level of competition. Competition answers will be scored in compliance with these rules for forms of answers.

Units of measurement are not required in answers, but they must be correct if given. When a problem asks for an answer expressed in a specific unit of measure or when a unit of measure is provided in the answer blank, equivalent answers expressed in other units are not acceptable. For example, if a problem asks for the number of ounces and 36 oz is the correct answer, 2 lbs 4 oz will not be accepted. If a problem asks for the number of cents and 25 cents is the correct answer, $0.25 will not be accepted.

The plural form of the units will always be provided in the answer blank, even if the answer appears to require the singular form of the units.

Geometric figures may not be drawn to scale and lengths of geometric figures should be assumed to be measured in "units" unless otherwise stated.

All answers must be expressed in simplest form. A "common fraction" is to be considered a fraction in the form $\pm\frac{a}{b}$, where a and b are natural numbers and GCF(a, b) = 1. In some cases the term "common fraction" is to be considered a fraction in the form $\frac{A}{B}$, where A and B are algebraic expressions and A and B do not share a common factor. A simplified "mixed number" ("mixed numeral," "mixed fraction") is to be considered a fraction in the form $\pm N\frac{a}{b}$, where N, a and b are natural numbers, $a < b$ and GCF(a, b) = 1. Examples:

Problem: What is $8 \div 12$ expressed as a common fraction? *Answer:* $\frac{2}{3}$ *Unacceptable:* $\frac{4}{6}$

Problem: What is $12 \div 8$ expressed as a common fraction? *Answer:* $\frac{3}{2}$ *Unacceptable:* $\frac{12}{8}$, $1\frac{1}{2}$

Problem: What is the sum of the lengths of the radius and the circumference of a circle with diameter $\frac{1}{4}$ unit expressed as a common fraction in terms of π? *Answer:* $\frac{1+2\pi}{8}$

Problem: What is $20 \div 12$ expressed as a mixed number? *Answer:* $1\frac{2}{3}$ *Unacceptable:* $1\frac{8}{12}$, $\frac{5}{3}$

Ratios should be expressed as simplified common fractions unless otherwise specified. Examples:

Acceptable Simplified Forms: $\frac{7}{2}$, $\frac{3}{\pi}$, $\frac{4-\pi}{6}$ *Unacceptable:* $3\frac{1}{2}$, $\frac{\frac{1}{4}}{3}$, 3.5, 2:1

Radicals must be simplified. A simplified radical must satisfy: 1) no radicands have a factor which possesses the root indicated by the index; 2) no radicands contain fractions; and 3) no radicals appear in the denominator of a fraction. Numbers with fractional exponents are *not* in radical form. Examples:

Problem: What is $\sqrt{15} \times \sqrt{5}$ expressed in simplest radical form? *Answer:* $5\sqrt{3}$ *Unacceptable:* $\sqrt{75}$

Answers to problems asking for a response in the form of a dollar amount or an unspecified monetary unit (e.g., "How many dollars...," "How much will it cost...," "What is the amount of interest...") should be expressed in the form ($) *a.bc* or *a.bc* (dollars), where *a* is an integer and *b* and *c* are digits. The *only* exceptions to this rule are when *a* is zero, in which case it may be omitted, or when *b* and *c* both are zero, in which case they both may be omitted. Answers in the form ($) *a.bc* or *a.bc* (dollars) should be rounded to the nearest cent unless otherwise specified. Examples:

Acceptable Forms: 2.35, 0.38, .38, 5.00, 5 *Unacceptable:* 4.9, 8.0

Do not make approximations for numbers (e.g., π, $\frac{2}{3}$, $5\sqrt{3}$) in the data given or in solutions unless the problem says to do so.

Do not perform any intermediate rounding (other than the "rounding" a calculator does) when calculating solutions. All rounding should be done at the end of the computation process.

Scientific notation should be expressed in the form $a \times 10^n$ where a is a decimal, $1 \le |a| < 10$, and n is an integer. Examples:

Problem: What is 6895 expressed in scientific notation? *Answer:* 6.895×10^3

Problem: What is 40,000 expressed in scientific notation? *Answer:* 4×10^4 *or* 4.0×10^4

An answer expressed to a greater or lesser degree of accuracy than called for in the problem will not be accepted. Whole-number answers should be expressed in their whole-number form. Thus, 25.0 will not be accepted for 25, and 25 will not be accepted for 25.0.