MATHCOUNTS®

2026 Chapter Competition Solutions

Are you wondering how we could have possibly thought that a Mathlete® would be able to answer a particular Sprint Round problem without a calculator?

Are you wondering how we could have possibly thought that a Mathlete would be able to answer a particular Target Round problem in less 3 minutes?

Are you wondering how we could have possibly thought that a particular Team Round problem would be solved by a team of only four Mathletes?

The following pages provide detailed solutions to the Sprint, Target and Team Rounds of the 2026 MATHCOUNTS Chapter Competition. These solutions show creative and concise ways of solving the problems from the competition.

There are certainly numerous other solutions that also lead to the correct answer, some even more creative and more concise!

We encourage you to find a variety of approaches to solving these fun and challenging MATHCOUNTS problems.

Special thanks to solutions author
Howard Ludwig
for graciously and voluntarily sharing his solutions with the MATHCOUNTS community.

Sprint 1

$(7 \times 2) - (3 \times 4 + 2) = (14) - (12 + 2) = 14 - 14 = \mathbf{0}$.

Sprint 2

$5\ \frac{\text{choc}}{\cancel{\text{mate}}} \times 5\ \cancel{\text{mate}} - 3\ \frac{\text{choc}}{\cancel{\text{mate}}} \times 5\ \cancel{\text{mate}} = (5 - 3)5\ \text{choc} = (2)5\ \text{choc} = \mathbf{10}\ \text{choc}$.

Sprint 3

All digits must be 2, 3, 5, or 7. A 1-digit perfect square cannot be prime, so the number must have at least 2 digits and value in 20s or higher. There is one such perfect square in the 20s: $5^2 = \mathbf{25}$.

Sprint 4

The two acute angles of a right triangle are complementary, so $y = 90° - 60° = 30°$. The two adjacent angles, one measuring 60° and the other measuring x, along and on the same side of ray $\overrightarrow{\mathrm{AC}}$ are supplementary, so $x = 180° - 60° = 120°$. Therefore, $x + y = 120° + 30° = \mathbf{150}°$.

Sprint 5

Lines of symmetry for a regular polygon must pass through the circumcenter and incenter (which coincide) of the polygon. For regular polygons with an even number of vertices, which includes octagons due to having 8 vertices, any line passing through a vertex and the incenter must pass through the diametrically opposite vertex. Thus, the vertices work as pairs to form lines of symmetry. For the octagon, the 8 vertices form $\frac{8}{2} = \mathbf{4}$ such distinct pairs and lines of symmetry.

Sprint 6

$2 \# 3 = 2(2) + 3(3) = 4 + 9 = \mathbf{13}$.

Sprint 7

The area relationship of the two figures is $(4\ \text{cm})w = 2(8\ \text{cm})^2 = 128\ \text{cm}^2$, so $w = \frac{128\ \text{cm}^2}{4\ \text{cm}} = 32$ cm. Therefore, the perimeter of the rectangle is $2(4\ \text{cm} + w) = 2(4\ \text{cm} + 32\ \text{cm}) = 2(36\ \text{cm}) = \mathbf{72}$ cm.

Sprint 8

The sum of the relevant multiples of 3 is the arithmetic series $3 + 6 + 9 + 12 + 15 + 18 + 21 + 24$, which has 8 terms whose average is the average of the first and last terms, thus $\frac{(3+24)}{2} = \frac{27}{2}$. Therefore, the desired sum is $\frac{8 \times 27}{2} = 4 \times 27 = \mathbf{108}$.

Sprint 9

Given the three side lengths of a right triangle, the area is ½ of the product of the two shortest sides. Now, $\frac{10}{3} = \frac{20}{6} < \frac{21}{6} = \frac{7}{2} < \frac{29}{6}$, so the area is $\frac{1}{2}\left(\frac{10}{3}\ \text{in}\right)\left(\frac{7}{2}\ \text{in}\right) = \left(\frac{5}{3}\right)\left(\frac{7}{2}\right)\ \text{in}^2 = \mathbf{\frac{35}{6}}\ \text{in}^2$.

Sprint 10

The area enclosed by a circle of radius r is πr^2. Read the question carefully to observe that we are not given the radius but the diameter, which is 2 times the radius. Thus, the outer circle has radius $\frac{10\ \text{ft}}{2} = 5$ ft and the inner circle has radius $\frac{6\ \text{ft}}{2} = 3$ ft. Therefore, the shaded area is the area enclosed by the outer circle minus the area enclosed by the inner circle, thus $\pi(5\ \text{ft})^2 - \pi(3\ \text{ft})^2 = (25 - 9)\pi\ \text{ft}^2 = \mathbf{16\pi}\ \text{ft}^2$.

Sprint 11
Addison's age is $31-6=25$ greater than Madison's, so the 2:1 ratio will occur when Addison is $2\times 25=50$ and Madison is 25, which is $25-6=\mathbf{19}$ years away.

Sprint 12
A loss of 15 from x corresponds to a relative loss of $100\ \%-80\ \%=20\ \%$, so a gain of 15 above x must be a relative gain of 20 %, so $x+15=\mathbf{120}\ \%\cdot x$.

Sprint 13
Pay at Al's is a fixed \$250; to earn the same amount at Carl's requires 10 h to reach \$150 and then $\frac{\cancel{\$}100}{\cancel{\$}25/\mathrm{h}}=4\ \mathrm{h}$ to get the remaining \$100 at \$25/h. Therefore, Charlene wishes to work 14 h each week, resulting in pay of $\frac{\$20}{\cancel{\mathrm{h}}}\times 14\ \cancel{\mathrm{h}}=\$\mathbf{280}\ or\ \$\mathbf{280.00}$ at Bertha's.

Sprint 14
$\frac{B}{A}=\frac{(25\ \cancel{\mathrm{m}})^2}{(75\ \cancel{\mathrm{m}})^2}=\left(\frac{1}{3}\right)^2=\mathbf{\frac{1}{9}}$.

Sprint 15
To go around one full cycle requires accumulating 360° of turns. At 15° per turn, one cycle requires $\frac{360°}{15°/\mathrm{turn}}=24$ turns. A segment for each turn having length 1 m requires pushing **24** m.

Sprint 16
The revenue for each cake is $\frac{\cancel{\$}12}{\cancel{\$}10}=1.2$ times as much as that for each pie. For equal revenue, the number of pies sold [240] must be 1.2 times the number of cakes sold, so the number of cakes is $\frac{240}{1.2}=\frac{240\times 10}{12}=20\times 10=\mathbf{200}$.

Sprint 17
The given sum of two side lengths is 20, which is greater than $\frac{1}{2}$ the perimeter, so the sides measured must be the two longest [opposite] sides, so each of the two shortest sides must have length $\frac{26-20}{2}=3$, while each longest side must be $\frac{20}{2}=10$. Therefore, the area of the rectangle is $10\times 3=\mathbf{30}$.

Sprint 18
For a product of 3 digits to be 18, the digits must be 1,2,9 with $3!=6$ permutations, 1,3,6 with 6 permutations, and 2,3,3 with $\frac{3!}{1!\,2!}=3$ permutations. That yields $6+6+3=\mathbf{15}$ qualifying integers.

Sprint 19
The first term is $a+0d=26$ and the twenty-first term is $a+20d=-4$, so $20d=-4-26=-30$. Thus, $d=-\frac{3}{2}$. We want the greatest integer n such that $a+(n-1)d>0$, thus $26-\frac{3}{2}(n-1)=\frac{55}{2}-\frac{3}{2}n>0$. Then $55>3n$ and $n<18\frac{1}{3}$. The greatest such integer n is **18**.

Sprint 20
$A=\frac{d_1 d_2}{2}$, so $d_2=\frac{2A}{d_1}=\frac{2\times 60\ \mathrm{in}^2}{6\ \mathrm{in}}=20\ \mathrm{in}$. The length of a side is $\sqrt{\left(\frac{d_1}{2}\right)^2+\left(\frac{d_2}{2}\right)^2}=\sqrt{(3\ \mathrm{in})^2+(10\ \mathrm{in})^2}=\sqrt{9+100}\ \mathrm{in}=\sqrt{\mathbf{109}}\ \mathrm{in}$.

Sprint 21

There are $2^5 = 32$ distinct sequences of toss results for 5 tosses. We do not want sequences with two or more consecutive tosses having the same result. In other words, we do not want sequences with results alternating back and forth through the sequence, thus not `HTHTH` nor `THTHT`. All 30 remaining distinct sequences meet the success criteria, so the desired probability is $\frac{30}{32} = \mathbf{\frac{15}{16}}$.

Sprint 22

The only difference between the two scenarios is one elevator taking 15 s to go 26 fl − 20 fl = 6 fl versus the second elevator taking 1 min = 60 s, which is 60 s − 15 s = 45 s longer to go 35 fl − 2 fl = 33 fl, which is 33 fl − 6 fl = 27 fl greater distance. The elevator speed when moving is, therefore, $\frac{27\text{ fl}}{45\text{ s}} = \mathbf{\frac{3}{5}}\ \frac{\text{fl}}{\text{s}}$. [The author of these solutions saw in one competition scoring room many answers of $\frac{5}{3}$, the reciprocal of the correct answer. Diligent use and algebraic handling of units of measurement with the numbers, as done throughout these solutions, catches most such errors. ☺]

Sprint 23

There are ${}_5\mathrm{C}_k$ arrangements of 5 people with k staying and $5 - k$ going, and we want to sum over $1 \le k \le 4$; this is equal to the even easier $2^5 - {}_5\mathrm{C}_0 - {}_5\mathrm{C}_5 = 32 - 1 - 1 = \mathbf{30}$.

Sprint 24

The sum of the 2 values $\frac{1}{a}$ and $\frac{1}{b}$ is given to be $\frac{2}{3}$, so the mean of the 2 values is $\frac{2/3}{2} = \frac{1}{3}$. Thus, one of $\frac{1}{a}$ and $\frac{1}{b}$ must be at least $\frac{1}{3}$ and the other must be at most $\frac{1}{3}$, with both being given as positive. In other words, one of a and b must be at most 3 and the other must be at least 3. Because [1] we want products of a and b and multiplication of integers is commutative, $ab = ba$ and we want distinct values of the product and [2] the given expression $\frac{1}{a} + \frac{1}{b}$ is symmetric in a and b, we can restrict consideration to $0 < a \le 3 \le b$. Right there we have $a = b = 3$ as a solution without even trying yet. We are given that there are two solutions, only one remains to be found and the only options left are integers a satisfying $0 < a < 3$, so a must be one of the two values 1 or 2. For that simple situation, there is no need to try to speed things up by taking the trouble to isolate b in terms of a—just plug and grind at most two values. Since we know 3 works, let's work our way down from 3 toward 0. For $a = 2$, $\frac{1}{b} = \frac{2}{3} - \frac{1}{2} = \frac{4-3}{6} = \frac{1}{6}$ so $b = 6$ works with $a = 2$. We now have two solutions, so none remain—no need to try $a = 1$. The sum of the solution products is $3 \times 3 + 2 \times 6 = \mathbf{21}$.

Sprint 25

Let N be the total number of students. Then $\frac{1}{2}N$ play piano and $\frac{1}{3}N$ play cello; because 3 students play both, we must subtract 3 to avoid double counting, and then the 20 who play neither need to be added to the total count, so $N = \frac{1}{2}N + \frac{2}{3}N - 3 + 20 = \frac{5}{6}N + 17$. Therefore, $\frac{1}{6}N = 17$, so $N = 6 \times 17 = \mathbf{102}$.

Sprint 26

For the quadratic equation $c_2x^2 + c_1x + c_0 = 0$ with roots a and b, Viète's formulas tell us that $a + b = -\frac{c_1}{c_2}$ and $ab = \frac{c_0}{c_2}$, so $a + b - ab = \frac{-c_1}{c_2} - \frac{c_0}{c_2} = \frac{-c_1 - c_0}{c_2}$. For this problem, $c_0 = -10$, $c_1 = 11$, and $c_2 = 6$, so $a + b - ab = \frac{-11+10}{6} = -\mathbf{\frac{1}{6}}$.

Sprint 27

The frog starts at (1; 1), a point of the form (o; o) [o for odd], and there are only 2 possible steps—to (o+1; o) and (o; o+1), both acceptable with 1 odd and 1 even coordinate. If the frog then switches direction, then 1 is added to the component with value o—except that unacceptably results in both coordinates being even, so we must continue in the same direction, thus adding 1 to o+1 to get both coordinates odd, so we are back to the starting condition. In essence, being at (o; o) requires us to go either (o; o)→(o+1; o)→(o+2; o) or (o; o)→(o; o+1)→(o; o+2), thus only 2 choices for a pair of steps together starting at (o; o). In essence, instead of having freedom to go 10 single steps up and 10 single steps right in any order, the second step must be in the same direction as the first; the third step can be either direction, but the fourth must match the third, and so on, so that we are making only 5 double steps up and 5 double steps right. Thus, we have altogether 5 indistinguishable UU steps and 5 indistinguishable RR steps in any order. How many distinct orders? $\frac{(5+5)!}{5!\,5!} = \frac{10\times9\times\cancel{8\times}7\times6}{5\times\cancel{4\times3\times2\times1}} = 2\times3\times7\times6 = \mathbf{252}$.

Sprint 28

Let S be the area of the shaded region, equal to $100\sqrt{3}$, and U be the area of the unshaded region within square ABCD. Let L be the length of a side of square ABCD and l be the length of a side of the inner square and of the shaded triangles. $U = L^2 - S$. Because the four shaded triangles are congruent and equilateral, each shaded triangle has area expressible as $\frac{S}{4}$ and $\frac{\sqrt{3}}{4}l^2$, so $100\sqrt{3} = S = \sqrt{3}l^2$, so $l^2 = 100$ and $l = 10$. Now, $L^2 = \frac{1}{2}(\mathrm{AC})^2$, where AC is twice the altitude of a shaded triangle plus l, thus, $\mathrm{AC} = 2\times\frac{1}{2}\sqrt{3}l + l = \left(\sqrt{3}+1\right)l = 10\left(\sqrt{3}+1\right)$, so $L^2 = \frac{1}{2}\left[10\left(\sqrt{3}+1\right)\right]^2 = 50\left(\sqrt{3}+1\right)^2 = 50\left(4+2\sqrt{3}\right) = 200 + 100\sqrt{3}$. Thus, $U = L^2 - S = \left(200+100\sqrt{3}\right) - 100\sqrt{3} = \mathbf{200}$.

Sprint 29

Consider four categories of integers in the span 1 .. 2026:

1. 1 .. 99: Any one digit in any of these values corresponds to at least half of the digits in the value, so is dominant. All 99 such values qualify as having a dominant digit.
2. 100 .. 999: If any two of the three digits in a value match, the value qualifies, but counting these involves multiple subcategories. The non-qualifying values are equivalent to those values that have all digits being different—no matches—which are very easy to count: There are 9 options (1 .. 9, as 0 does not work for a 3-digit number) for the 100s digit, 9 options left (bringing in 0 but dropping the 100s digit) for the 10s digit, and 8 options left for the 1s digit. Thus, $9\times9\times8 = 648$ of the $999 - 99 = 900$ candidates fail to qualify, so $900 - 648 = 252$ qualify.
3. 1000 .. 1999: Like category 2, having any two digits match makes that digit value dominant, so we will again count all of the values in the span that that have all digits being different. The 1000s digit has only the 1 option (1), so there are 9 options left (all other than 1) for the 100s digit, 8 options left for the 10s digit, and 7 options left for the 1s digit. Thus, $1\times9\times8\times7 = 504$ of the $1999 - 999 = 1000$ possibilities fail to qualify, leaving $1000 - 504 = 496$ to qualify.
4. 2000 .. 2026: At least half of the digits in each of 2000 .. 2010 are 0, making 0 the dominant digit and qualifying these 11 values. Then 2011 and 2012 have half 1s and half 2s, respectively, for 2 more qualifying values. At least half of the digits in each of 2020 .. 2026 are 2, making 2 the dominant digit and qualifying these 7 values, so $11 + 2 + 7 = 20$ values in this category qualify.

In total, there are $99 + 252 + 496 + 20 = \mathbf{867}$ qualifying values.

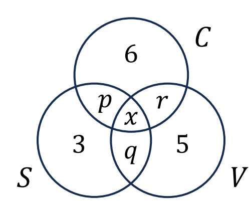
Sprint 30

Square both sides of $\sqrt{x^2+\sqrt{x}}+\sqrt{x^2-\sqrt{x}}=2$: $x^2+\sqrt{x}+x^2-\sqrt{x}+2\sqrt{x^4-x}=4$, which simplifies to $2x^2+2\sqrt{x^4-x}=4$. Now divide both sides by 2 and shift the resulting x^2 term to the right side: $\sqrt{x^4-x}=2-x^2$. Square both sides again: $x^4-x=4-4x^2+x^4$. Shift everything to the left side and cancel the quartic terms, leaving the quadratic equation $4x^2-x-4=0$, which can be solved using the quadratic formula: $x=\frac{-(-1)\pm\sqrt{(-1)^2-4(4)(-4)}}{2(4)}=\frac{1\pm\sqrt{65}}{8}$. Having squared both sides twice provides opportunity for extraneous roots. The form of the potential answers as the sum of a rational part and an irrational part makes for a challenge to verify versus disqualify the candidates as valid roots by substituting the candidates into the given equation with a pair of nested square roots in a Sprint Round with no calculator support. However, we can readily assess the qualitative behavior of the left side of the given equation adequately to accept one candidate root and reject the other. The two occurrences of $\sqrt{x}$ in the given equation forces $x\ge 0$. Because $\sqrt{65}>8>1$, we have $\frac{1-\sqrt{65}}{8}<0$, thus disqualifying that candidate. We see also that $\frac{1+\sqrt{65}}{8}>1$. If we substitute 1 for x on the left side of the equation, we get $\sqrt{1+1}+\sqrt{1-1}=\sqrt{2}+0=\sqrt{2}$, which is less than the 2 on the right side. We can also see that as we increase x above 1, the two main square roots on the left side, and thus their sum, keep increasing arbitrarily large, and thus, must pass through 2. Thus, there must be a root, and the only remaining candidate is $\frac{1+\sqrt{65}}{8}$, so $x=\frac{1+\sqrt{65}}{8}$ is the one and only solution. Therefore, $8x=\mathbf{1+\sqrt{65}}$, which may be expressed instead as $\mathbf{\sqrt{65}+1}$—either answer is acceptable.

Target 1

Let p be the regular price of the game. The payment was $1p+(1-0.20)p=1.8p=\$27$. Therefore, $p=\frac{\$27}{1.8}=\$\mathbf{15}$ or $\$\mathbf{15.00}$—either answer is acceptable.

Target 2



There are 24 children, of whom $6+3+5=14$ have been accounted for in the chocolate only, strawberry only, and vanilla only regions. The other children are in the regions notated by the variables p, q, r, and x. We seek the value of x, the count of those who like all three flavors, that is those who are elements of $C\cap S\cap V$. We are given:

[1] $p+x=5; q+x=4; r+x=7.$ Given in question.

[2] $p+q+r+x=24-14=10.$ Children unaccounted for.

Adding the three equations in [1] yields [3] $p+q+r+3x=16$.

Subtracting [3] − [2] yields: $2x=6$, so $x=\mathbf{3}$.

Target 3

The area of rectangle is $10\times(2+4)=60$. There are five shaded triangles whose bases are on $\overline{\text{AE}}$ and have a total width of 8, and the height of each is 2, so the total area of the shaded triangles is $\frac{1}{2}\times 8\times 2=8$. Therefore, the fraction of DEAR that is shaded is $\frac{8/4}{60/4}=\mathbf{\frac{2}{15}}$.

Target 4

If the amount of paint was unlimited, but still restricted to 3 colors, then each of the 4 walls could be any of the 3 colors yielding $3^4 = 81$ distinct configurations, each in 1 of 4 categories:

1. All 4 walls of the same color;
2. 3 walls of one color and 1 wall of a second color;
3. 2 walls of one color and 2 walls of a second color;
4. 2 walls of one color, 1 of a second color, and 1 of a third color.

However, we do have restrictions on amount of paint, so that no configuration in categories 1 and 2 is achievable, whereas every configuration in categories 3 and 4 is achievable. Counting configurations in categories 1 and 2 is very easy, whereas having two colors each covering the same nonzero count of walls as in configurations 3 and 4 gets a bit trickier and more prone to mistake. Therefore, let's use the complement approach and subtract the configuration counts in the two unachievable categories [1 and 2] from the total count of 81. For category 1, all walls are of any 1 of the 3 colors for 3 configurations. For category 2, any 1 of the 4 walls may be painted any 1 of the 3 colors, and all 3 remaining walls are painted a second color, of which 2 choices remain, yielding $4 \times 3 \times 2 = 24$ configurations. Therefore, $81 - 3 - 24 = \mathbf{54}$ configurations are achievable.

Target 5

The possible configurations satisfying the stated criteria are:

1. 8-8-4, with $\frac{3!}{2!\,1!} = 3$ permutations;
2. 8-7-5, with $\frac{3!}{1!\,1!\,1!} = 6$ permutations;
3. 8-6-6, with $\frac{3!}{1!\,2!} = 3$ permutations;
4. 7-7-6, with $\frac{3!}{2!\,1!} = 3$ permutations.

Each permutation is a qualifying configuration, so the total number of configurations is $3 + 6 + 3 + 3 = \mathbf{15}$.

Target 6

The perimeter 4 in consists of 4 congruent sides, so each side has length $\frac{4\text{ in}}{4} = 1$ in. Let E be the point of intersection of the two diagonals; E is the right-angle vertex for each of four congruent triangles. Thus, $\text{AE} = \text{EC} = \frac{1}{2}\,\text{AC} = \text{BD} = 2\,\text{BE} = 2\,\text{ED}$; for $\triangle\text{ABE}$, we have $\text{BE} : \text{AE} : \text{AB} = 1 : 2 : \sqrt{5}$. Because $\text{AB} = 1$ in, we thus have $\text{BE} = \frac{1\text{ in}}{\sqrt{5}}$, $\text{AE} = \frac{2\text{ in}}{\sqrt{5}}$, so the desired area is given by $\text{Area(ABCD)} = 4\,\text{Area(ABE)} = 4 \times \frac{1}{2}\,\text{AE} \cdot \text{BE} = 2\,\text{AE} \cdot \text{BE} = 2 \times \frac{1\text{ in}}{\sqrt{5}} \times \frac{2\text{ in}}{\sqrt{5}} = \mathbf{\frac{4}{5}}\text{ in}^2$.

Target 7

Let $\bar{v}$ be the average speed over the trip. We must be careful to remember that average speed of a trip composed of segments of equal distance is **not** the arithmetic mean of the segment speeds but rather the harmonic mean of the segment speeds (i.e. the quotient of the total distance divided by the total time). Therefore, $\bar{v} = \frac{740\text{ km}+740\text{ km}}{8.5\text{ h}+10\text{ h}} = \frac{1480\text{ km}}{18.5\text{ h}} = 80\ \frac{\text{km}}{\text{h}}$. However, the question states that the answer is to be expressed in terms of $\frac{\text{m}}{\text{s}}$ (i.e., the coherent SI unit of speed). The kilometer and hour that we are given are defined in terms of the desired meter and second, respectively, as $\text{km} = 1000$ m and $\text{h} = 3600$ s, so $\bar{v} = 80\ \frac{\text{km}}{\text{h}} = 80 \times \frac{10\cancel{00}\text{ m}}{36\cancel{00}\text{ s}} = 22.222\ldots\ \frac{\text{m}}{\text{s}}$, which rounds, as instructed, to the nearest $0.1\ \frac{\text{m}}{\text{s}}$ as $\mathbf{22.2}\ \frac{\text{m}}{\text{s}}$.

Target 8

Let's number the seats from 1 to 6 in clockwise order around the table with Greg sitting between 6 and 1. Let's consider 3 cases:

1. The together pair are in seats 1 and 2, or the equivalent mirror-image situation of seats 5 and 6—work out the 1 & 2 case, then multiply by 2 to handle that case and the 5 & 6 case. Any of the 6 members other than Greg can go in seat 1 and then the 1 pair-partner must be in seat 2. Any of the remaining 4 can go in seat 3; either of the 2 in the only not-yet-used pair can go in seat 4, then the 1 remaining member of pair 2 can go in seat 5, and the 1 remaining person can go in seat 6. This case provides $2 \times 6 \times 1 \times 4 \times 2 \times 1 \times 1 = 96$ seating layouts.
2. The together pair are in seats 2 and 3, or the equivalent mirror-image situation of seats 4 and 5—work out the 2 & 3 case, then multiply by 2 to handle that case and the 4 & 5 case. Any of the 6 members other than Greg can go in seat 2 and then the 1 pair-partner must be in seat 3. Any of the remaining 4 can go in seat 1, with the 1 partner going to seat 5 in order to split the third pair is seats 4 & 6, with 2 choices for seat 4 and the 1 remaining having to go in seat 6. This case provides $2 \times 6 \times 1 \times 4 \times 1 \times 2 \times 1 = 96$ seating layouts.
3. The together pair are in seats 3 and 4. There is not any equivalent mirror-image situation. Any of the 6 members other than Greg can go in seat 3 and then the 1 pair-partner must be in seat 4. Any of the remaining 4 can go in seat 1, and either of the 2 from the only not-yet-used pair can go in seat 2. Finally, either of the remaining 2 can go in seat 5 and the remaining 1 in seat 6. This case provides $6 \times 1 \times 4 \times 2 \times 2 \times 1 = 96$ seating layouts.

Therefore, the total number of qualifying seating arrangements is $96 + 96 + 96 = \mathbf{288}$.

Team 1

[The official currency exchange symbols for the American dollar and Canadian dollar are USD and CAD, respectively.] The range is the maximum value minus the minimum value; therefore,
$1500\,\cancel{\text{USD}} \times 1.39\,\frac{\text{CAD}}{\cancel{\text{USD}}} - 1500\,\cancel{\text{USD}} \times 1.33\,\frac{\text{CAD}}{\cancel{\text{USD}}} = 1500(1.39 - 1.33)\text{ CAD} = 1500 \times 0.06\text{ CAD} = 15 \times 6\text{ CAD} = \mathbf{90}\text{ CAD}$.

Team 2

$540° = (x + 15°) + (2x + 25°) + (3x - 20°) + (4x + 30°) + (5x + 10°) = 15x + 60°$, so $15x = 540° - 60° = 480°$. Thus, $x = \frac{480°}{15} = 32°$. Now, $x < 2x < 4x$ and $15° < 25° < 30°$, so $x + 15° < 2x + 25° < 4x + 30°$. Then $3x < 4x$ and $-20° < 30°$, so $3x - 20° < 4x + 30°$. Thus, the maximum is either $4x + 30°$ or $5x + 10°$—the former is $4 \times 32° + 30° = 158°$, and the latter is $5 \times 32° + 10° = 170°$. Therefore, the greatest angle is $\mathbf{170°}$.

Team 3

$\frac{V_X}{V_Y} = \frac{\pi r_X^2 h_X}{\pi r_Y^2 h_Y} = \left(\frac{r_X}{r_Y}\right)^2 \left(\frac{h_X}{h_Y}\right)$. Now, $h_Y = 3h_X$ and $r_Y = \frac{1}{2} r_X$, so $\frac{r_X}{r_Y} = 2$ and $\frac{h_X}{h_Y} = \frac{1}{3}$, so $\frac{V_X}{V_Y} = 2^2 \times \frac{1}{3} = \mathbf{\frac{4}{3}}$.

Team 4

Case	Number of permutations
6-6-6-6-6	$1 \times \frac{5!}{5!} = 1$
6-6-6-6-(2,3,4,5)	$4 \times \frac{5!}{4!\,1!} = 20$
6-6-6-5-5	$1 \times \frac{5!}{3!\,2!} = 10$
6-6-6-5-(3,4)	$2 \times \frac{5!}{3!\,1!\,1!} = 40$
6-6-6-4-4	$1 \times \frac{5!}{3!\,2!} = 10$
6-6-5-5-5	$1 \times \frac{5!}{2!\,3!} = 10$
6-6-5-5-4	$1 \times \frac{5!}{2!\,2!\,1!} = 30$
6-5-5-5-5	$1 \times \frac{5!}{1!\,4!} = 5$
Total	$= \mathbf{126}$

Team 5

The formula for the total revenue is $R = n_c p_c + n_a p_a + n_s p_s$, where n is the number of tickets sold and p is the price of a ticket, with the subscripts c, a, and s representing the category child, adult, and senior, respectively. The graph yields $n_c = 3, n_a = 5 + 4 + 4 + 4 + 4 = 21, n_s = 4 + 4 + 1 = 9$. Let C be the so far unknown price of a child ticket, the value of which is the goal of the problem. Then $p_c = C$; $p_c = \frac{1}{2} p_s$ so $p_s = 2p_c = 2C$; $p_s = \frac{1}{2} p_a$ so $p_a = 2p_s = 2(2C) = 4C$. Thus, $\$840 = R = 3(C) + 21(4C) + 9(2C) = 3C + 84C + 18C = 105C$. Therefore, $C = \frac{\$840}{105} = \$\mathbf{8}$ *or* $\$\mathbf{8.00}$.

Team 6

Water needs multiplying by 2 with water usage per acre unchanged means that the acreage (land area) is being multiplied by the same factor of 2. Thus, land area $A = lw$ goes to $2lw = (kl)(kw) = k^2 lw$, which means $k^2 = 2$, so the each of the length and width are multiplied by $\sqrt{2} = 1.414\,21\ldots$. Therefore, the relative increase in each of the length and width of the field is $1.414\,21\ldots - 1 = 0.414\,21\ldots \times 100\ \% = 41.421\ldots\ \%$, which rounds to the nearest 0.1 % as **41.4** %.

Team 7

A 2 lb bag of mix contains 1 lb of chocolate, which costs $\frac{\$12}{3\ \cancel{\text{lb}}} \times 1\ \cancel{\text{lb}} = \$4$, and 1 lb of peanuts, which costs $\frac{\$18}{6\ \cancel{\text{lb}}} \times 1\ \cancel{\text{lb}} = \$3$. Thus, the total ingredient cost for a bag is $\$4 + \$3 = \$7 = 0.8p$, where p is the desired selling price for each bag of mix. Therefore, $p = \frac{\$7}{0.8} = \$\mathbf{8.75}$.

Team 8

For 2 days, each independently at $20\ \% = \frac{20/20}{100/20} = \frac{1}{5}$ for jeans means $5\text{ d} - 2\text{ d} = 3\text{ d}$, each independently at $1 - \frac{1}{5} = \frac{4}{5}$ for non-jeans. This is a binomial distribution, so the requested probability is $\frac{5!}{2!\,3!}\left(\frac{1}{5}\right)^2\left(\frac{4}{5}\right)^3 = \frac{(\cancel{5\times}4)\times(1)\times(4\times4\times4)}{(2)\times(\cancel{5\times}5)\times(5\times5\times5)} = \frac{2\times4^3}{5^4} = \mathbf{\frac{128}{625}}$.

Team 9

We have a finite sequence consisting of a 1 followed by a subsequence of a nonnegative integer n count of terms each of which is a 1, followed by a subsequence of a nonnegative integer m count of terms each of which is a 2, followed by a subsequence of a nonnegative integer $2024 - n - m$ count of terms each of which is a 3, followed by a 3. Thus, $0 \le \min(m, n) \le \max(m, n) \le n + m \le 2024$ in order to be meaningful and totals $1 + n + m + (2024 - n - m) + 1 = 2026$ terms in the sequence, as required by the question. We now focus on the middle 2024 terms because those are the only ones that can change to produce a different sequence—the term at each end is fixed in value. Let's put a vertical boundary bar immediately to the right of the latest term with value 1 in the full 2026-term sequence and another such bar immediately to the left of the earliest term with value 3. The two bars will be contiguous if there are no terms with value 2. A bar will be at the far left end of the subsequence of the middle 2024 terms if there are no 1s in the full sequence except the fixed initial 1, and, similarly, a bar will be at the far right end of the subsequence of the middle 2024 terms if there are no 3s in the full sequence except the fixed initial 3. Moving a bar to another location means the sequence has changed, which counts as a distinct sequence. In essence, we have 2024 slots for numbers (not counting the fixed number values at each end) and 2 slots for boundary bars—with all 1s to the left of the leftmost bar, all 2s between the two bars, and all 3s to the right of the rightmost bar. One boundary bar can go in any of the 2026 slots and the other boundary bar can go in any of the remaining 2025 slots; however, swapping the two bars does not change the sequence, so we are double-counting, which we need to remedy. Therefore, the count of distinct sequences is $\frac{2026 \times 2025}{2} = \mathbf{2\ 051\ 325}$.

Team 10

The figure shows that opposite sides of the hexagon are separated by the same distance as the diameter of the circles, which is 2. For a regular hexagon, like ours, this separation is also equal to $\sqrt{3}s$, where s is the length of each side of the hexagon. Thus, we have $s = \frac{2}{\sqrt{3}}$—do not waste time and effort "normalizing" or "simplifying" radicals out of denominators in intermediate calculations as such actions are often counterproductive, like here. The area of a hexagon is $6 \times \frac{\sqrt{3}}{4} s^2 = 6 \times \frac{\sqrt{3}}{\cancel{4}} \times \frac{\cancel{4}}{3} = 2\sqrt{3}$. The hexagon cuts out a sector of 120° arc from each circle (because each interior angle of a regular hexagon has measure 120°, and the center of each circle is at the center of a circle). Thus, the sectors from the three circles combine to form one full circle of radius 1, so the interior of that circle has area π. Therefore, the shaded area is $2\sqrt{3} - \pi = 0.3225\ldots$, which rounds to the nearest hundredth as **0.32** *or* **.32**. [The explicit 0 for the integer part is considered good practice—often even required—by numerous organizations. MATHCOUNTS uses the explicit 0 in all printed materials but does not require it in a Mathlete's answers. The Texas UIL Number Sense Competition takes the contrary approach of forbidding explicitly writing an integer part that is 0.]