P U M ∴ C

Algebra B

1. Find the sum of the solutions to $\dfrac{1}{1+\dfrac{1}{|x-25|}} = \dfrac{49}{50}$.

2. For what value of n is $\dfrac{1}{2\cdot 5}+\dfrac{1}{5\cdot 8}+\dfrac{1}{8\cdot 11}+\ldots+\dfrac{1}{n(n+3)} = \dfrac{25}{154}$?

3. Let
$$a_k = 0.\overbrace{0\ldots0}^{k-1\ 0's}\,1\,\overbrace{0\ldots0}^{k-1\ 0's}\,1$$
The value of $\displaystyle\sum_{k=1}^{\infty} a_k$ can be expressed as a rational number $\dfrac{p}{q}$ in simplest form. Find $p+q$.

4. If $a_1, a_2, \ldots$ is a sequence of real numbers such that for all n,
$$\sum_{k=1}^{n} a_k \left(\frac{k}{n}\right)^2 = 1,$$
find the smallest n such that $a_n < \dfrac{1}{2018}$.

5. Let $x_0, x_1, \ldots$ be a sequence of real numbers such that $x_n = \dfrac{1+x_{n-1}}{x_{n-2}}$ for $n \geq 2$. Find the number of ordered pairs of positive integers (x_0, x_1) such that the sequence gives $x_{2018} = \dfrac{1}{1000}$.

6. Suppose real numbers a, b, c, d satisfy $a+b+c+d = 17$ and $ab+bc+cd+da = 46$. If the minimum possible value of $a^2+b^2+c^2+d^2$ can be expressed as a rational number $\dfrac{p}{q}$ in simplest form, find $p+q$.

7. For $k \in \{0, 1, \ldots, 9\}$, let $\epsilon_k \in \{-1, 1\}$. If the minimum possible value of $\displaystyle\sum_{i=1}^{9}\sum_{j=0}^{i-1} \epsilon_i \epsilon_j 2^{i+j}$ is m, find $|m|$.

8. Let a, b, c be non-zero real numbers that satisfy $\dfrac{1}{abc}+\dfrac{1}{a}+\dfrac{1}{c} = \dfrac{1}{b}$. The expression $\dfrac{4}{a^2+1}+\dfrac{4}{b^2+1}+\dfrac{7}{c^2+1}$ has a maximum value M. Find the sum of the numerator and denominator of the reduced form of M.