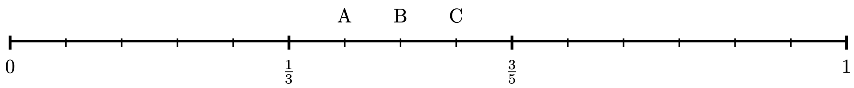
1. Ashley (A), Bailing (B), and Claire (C) stand equally-spaced on a number line between the points $\frac{1}{3}$ and $\frac{3}{5}$, as shown below. Claire's position is given by the fraction $\frac{n}{15}$. What is the value of n?



   Proposed by: Amber Bajaj

   Answer: $\boxed{8}$

   Solution: Bailing is halfway between $\frac{1}{3}$ and $\frac{3}{5}$. Taking the midpoint is the same as taking the average of these two points, so Bailing is at $\frac{\frac{1}{3}+\frac{3}{5}}{2} = \frac{1}{2} \times \frac{14}{15} = \frac{7}{15}$. Claire is halfway between Bailing and $\frac{3}{5}$, so her position is $\frac{\frac{7}{15}+\frac{3}{5}}{2} = \frac{1}{2} \times \frac{16}{15} = \frac{8}{15}$.

2. Bruno and Brunita are standing together in a line for a movie. They notice that the number of people ahead of them is seven times the number of people behind them. The total number of people in the line, including the two of them, is somewhere between 20 and 30. How many people are in the line?

   Proposed by: Amber Bajaj

   Answer: $\boxed{26}$

   Solution: Let x be the number of people behind Bruno and Brunita. Then, since there are seven times as many people in front of them as behind them, the total number of people in the line is $7x + 2 + x = 8x + 2$. Since x must be an integer, the only value of x for which $20 \leq 8x + 2 \leq 30$ is $x = 3$. Thus, the total number of people is $8(3) + 2 = 26$.

3. Bruno has some number of nickels (5 cents), dimes (10 cents), and quarters (25 cents), possibly zero of any type. If he has 30 cents total, how many possible configurations of coins can he have (coin ordering does not matter)?

   Proposed by: William Wang

   Answer: $\boxed{5}$

   Solution: Let the tuple (a, b, c) denote having a nickels, b dimes, c quarters. If Bruno has a quarter, then he must have a nickel to get a total of 30 cents. So $(1, 0, 1)$ is one configuration. If he has no quarter, then he can only have nickels and dimes. He can have either 0, 1, 2, or 3 dimes, giving 4 more configurations: $(6, 0, 0)$, $(4, 1, 0)$, $(2, 2, 0)$, $(0, 3, 0)$. Thus, there are a total of 5 configurations.

4. Lily and Coco each roll a fair 6-sided die. What is the probability that Lily's number is greater than Coco's number?

   Proposed by: Coco Zeng

   Answer: $\boxed{\frac{5}{12}}$

   Solution: The probability that Lily and Coco roll the same number is $\frac{1}{6}$, since Lily has a $\frac{1}{6}$ chance of rolling Coco's number. The probability that Lily's number is greater than Coco's number is the same as the probability that Coco's number is greater than Lily's number. By symmetry, the probability that Lily's number is greater than Coco's number is $(1 - \frac{1}{6})/2 = \frac{5}{12}$.

5. There are n red and m blue balls in a jar. If you increase the number of red balls by 20% of the total balls in the jar, then the number of red balls is 20% more than the number of blue balls. What is $\frac{n}{m}$?

   Proposed by: Alice Wang

   Answer: $\boxed{\frac{5}{6}}$

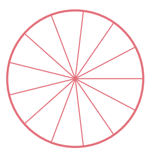
   Solution: We can express the situation as

$$n + 0.2(n+m) = 1.2m$$

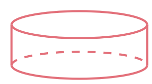
   Multiplying everything by 5, we have

$$\begin{aligned} 5n + n + m &= 6m \\ 6n &= 5m \\ \frac{n}{m} &= \frac{5}{6}. \end{aligned}$$

6. Peter has a cake in the shape of a cylinder with height 2 in and base radius 5 in. He cuts the cake into 13 equally sized wedges. By how many in^2 does the surface area of the cake increase after Peter has made these cuts?



Top view



Side view

   Proposed by: Ryan Lee

   Answer: $\boxed{260}$

   Solution: Each wedge creates two new rectangles with area $5 \cdot 2 = 10$, so in total we get an increase in area of $10 \cdot 2 \cdot 13 = 260$.

7. We have a list where every number n from 1 to 26 (inclusive) appears exactly n times (1 appears once, 2 appears twice, etc). What is the median of this list of numbers?

   Proposed by: Lily Bowen

   Answer: $\boxed{19}$

   Solution: This list contains $\frac{(1+26)(26)}{2} = 351$ elements, so the median is equal to the value of the 176th element (assuming the list is in increasing order). To figure out which number this is, we can observe that $\frac{(1+n)(n)}{2} = 171$ when $n = 18$, so the 172nd element is 19. Since this element occurs 19 times, the 176th element is also 19.

8. You are sending letters to friends A, B, and C. You have three letters, one written to each friend. Friends A, B, and C have a 50%, 40%, and 10% chance of responding if sent the right letter, respectively. All friends are guaranteed to respond if sent the wrong letter. You shuffle the letters and send one to each friend. What is the probability you get no response?

   Proposed by: Jonathan Xu

Answer: $\boxed{\dfrac{9}{200}}$

Solution: The only way to receive no responses is if all three friends get their correct letters. The probability of a perfect match is $\frac{1}{6}$, and then the chances of silence are $(1-0.5)(1-0.4)(1-0.1) = 0.27$. Hence the overall probability is $\frac{1}{6} \cdot 0.27 = \frac{9}{200}$.

9. The BrUMO staff distributes scratch paper to 13 teams for the team round contest, giving the same amount of paper to every team while giving out as much paper as possible. When the BrUMO staff have n sheets of paper, each team receives m pieces, and there are 3 leftover. If the BrUMO staff instead distributed $10n$ sheets, each team would receive 32 pieces of paper (with some leftover). What is the value of $n+m$?

   Proposed by: Amber Bajaj

   Answer: $\boxed{45}$

   Solution: Since there are 3 papers leftover after initially handing out n, we know $n = 13m+3$, where m is an integer (because m represents the number of papers each team receive). Multiplying by 10, we have $10n = 10(13m+3) = 130m+30$. Re-distributing, we have $130m+30 = 13(10m+2)+4$, meaning each of the 13 teams now receives $10m+2$ papers. We are given that each team receives 32, so $10m+2 = 32 \implies m = 3$. Substituting back into $n = 13m+3$, we have $n = 13(3)+3 = 42$. Thus, $n+m = 42+3 = 45$.

10. 15 friends want to play a game 3 times. Each game seats exactly 10 players, and the other 5 must sit out. They want to make sure everyone plays at least once. What is the smallest possible number of people who must play at least 2 games?

    Proposed by: Jonathan Xu

    Answer: $\boxed{8}$

    Solution: There are 30 total "seats" across the three games and 15 people. Giving everyone one game uses 15 seats, leaving 15 extra appearances that must be distributed. To minimize how many people play at least twice, pack these extras into as few people as possible by letting some play three times (each such person contributes 2 extras). Seven people playing three times supply 7 x 2 = 14 extras; you still need 1 more extra, which forces one additional person to play a second time. Thus, 7 people play three times and 1 person plays twice, totaling 8 people who play at least twice.

11. How many values of k is 20^{20} the least common multiple of $25^{10}, 10^{10}$, and k?

    Proposed by: Amy Qiao

    Answer: $\boxed{21}$

    Solution: We compute the prime factorizations $20^{20} = 2^{40} \cdot 5^{20}, 25^{10} = 5^{20}$, and $10^{10} = 2^{10} \cdot 5^{10}$. k must be in the form $k = 2^{k_1} \cdot 5^{k_2}$. Since $lcm(5^{20}, 2^{10} \cdot 5^{10}, 2^{k_1} \cdot 5^{k_2}) = 2^{40} \cdot 5^{20}$, we see that $k_1 = 40$ and k_2 can be any number in $\{0, 1, 2, \ldots, 20\}$. Therefore, there are 21 possible values of k.

12. Every iDevice has a three-digit positive integer (without leading zeros) as its password. Unfortunately, Uruno's iDevice is glitchy, and every time he logs in, the middle digit of his password gets replaced with the average of the first and third digits. Uruno wants to choose a password that won't change even after the glitch. How many possible passwords are there for Uruno?

Proposed by: Jaeho Lee

Answer: $\boxed{45}$

Solution: Let the three-digit password be $\overline{abc}$, which stands for $100a + 10b + c$. Since it's a three-digit positive integer, a must be a digit from 1 to 9, while b and c can be any digit from 0 to 9. The glitch replaces the middle digit b with the average of the first and third digits, which is $\frac{a+c}{2}$. For the password to remain unchanged, the original middle digit b must already be equal to this average. So, we are looking for the number of passwords that satisfy the equation $b = \frac{a+c}{2}$. Multiplying by 2, we get $2b = a + c$. This equation tells us that the sum $a + c$ must be an even number. This can only happen if a and c are both even, or if a and c are both odd. We can count these two cases.

First, let's count the case where a and c are both odd. The possible odd digits for a are $\{1, 3, 5, 7, 9\}$, which is 5 choices. The possible odd digits for c are also $\{1, 3, 5, 7, 9\}$, which is 5 choices. For any of these pairs, the sum $a + c$ is even, and $b = \frac{a+c}{2}$ will be a valid digit (from $b = \frac{1+1}{2} = 1$ to $b = \frac{9+9}{2} = 9$). So, there are $5 \times 5 = 25$ passwords in this case.

Second, let's count the case where a and c are both even. The possible even digits for a are $\{2, 4, 6, 8\}$ (4 choices, since a cannot be 0). The possible even digits for c are $\{0, 2, 4, 6, 8\}$ (5 choices). For any of these pairs, $b = \frac{a+c}{2}$ will also be a valid digit (from $b = \frac{2+0}{2} = 1$ to $b = \frac{8+8}{2} = 8$). So, there are $4 \times 5 = 20$ passwords in this case.

To find the total number of possible passwords, we add the two cases together: $25 + 20 = 45$.

13. A circular room has radius 10. In the center of the room stands a circular pillar of radius 5. You and a lamp are each placed independently and randomly on the outer wall of the room (that is, on the circumference). What is the probability that you have a direct line of sight to the lamp (meaning you can draw a straight line from the lamp to you)?

Proposed by: Jonathan Xu

Answer: $\boxed{\dfrac{2}{3}}$

Solution: Consider the right triangle formed by the center of the room, your position (distance $R = 10$ from center), and a tangent point on the pillar (distance $r = 5$ from center). Since the tangent is perpendicular to the radius at the touch point:

$$\sin(\alpha) = \frac{r}{R} = \frac{5}{10} = \frac{1}{2}$$

Therefore $\alpha = 30$. The two tangent lines create a blocked angular region. By symmetry about the line from you to the center, the blocked arc subtends an angle of:

$$\theta_{\text{blocked}} = 2(90 - \alpha) = 2(90 - 30) = 120$$

So, the probability of direct line of sight is:

$$P(\text{line of sight}) = \frac{360 - 120}{360} = \frac{240}{360} = \frac{2}{3}$$

14. In rectangle $ABCD$, let circle ω with center O outside $ABCD$ intersect the rectangle at points A and B. Suppose point P exists on side CD such that PA and PB are tangent to ω. If $\frac{OA}{AP} = \frac{1}{10}$, find $\frac{OA}{AD}$.

Proposed by: Faizaan Siddique

Answer: $\boxed{\frac{\sqrt{101}}{100}}$

Solution: First, observe that the perpendicular bisector of AB must pass through both points O and P. This is because $OA = OB$, so OAB is isosceles, and $PA = PB$ because PA and PB are tangents to the same circle from the same external point P. Thus, PAB is also isosceles. Next, let E be the foot of the altitude of O onto AB; E is the midpoint of AB. Notice that $AD = EP$ since E is the midpoint of AB and P is the midpoint of CD. Thus, $\frac{OA}{EP} = \frac{OA}{AD}$. Additionally, $\triangle OAP \sim \triangle AEP$ because both are right triangles that share $\angle OPA = \angle APE$ so $\frac{AE}{EP} = \frac{OA}{AP} = \frac{1}{10}$. Then, if we set $OA = x$ and $EP = y$, then applying the Pythagorean Theorem to triangle AEP gives:

$$(y)^2 + \left(\frac{y}{10}\right)^2 = (10x)^2 \implies y = \frac{100}{\sqrt{101}}x.$$

We can now evaluate our answer:

$$\frac{OA}{AD} = \frac{OA}{EP} = \frac{x}{\frac{100}{\sqrt{101}}x} = \boxed{\frac{\sqrt{101}}{100}}.$$

15. Ryan starts with an even two-digit positive integer (leading zeros are not allowed). He writes this number down and then repeatedly performs both steps in order:

    1. Divide the current number by 2. If the result is an integer, write it down. Otherwise, the process stops.
    2. Choose one digit of this number and replace it with the digit 8. Write down the result.

    At the end of the process, Ryan has written down k distinct numbers. What is the greatest possible value of k?

    Proposed by: Mason Yu

    Answer: $\boxed{16}$

    Solution: Draw a graph between every 2 digit number with 8 in one of its digits (which represents changing one of the digits to 8). We have that $84, 48, 82, 28$ form cycles between them, 18 ends the chain of 2 digit numbers, $78, 38$ never enter this loop, and $68, 86, 88, 58, 98$ enter the cycle. Also, 28 can lead to 18, which leads to 8.

    So to cover as many numbers in the graph, we take the following: $88 \to 84 \to 82 \to 48 \to 28 \to 18 \to 8$. We can have two extra numbers at the very beginning to get to the 88. For example, we have 76, and then dividing gives 38, which replaces the first digit to get 88.

    So we have 7 numbers connected. This means that we can take the two numbers at the beginning, then add $7 \cdot 2$ because you can write down each number as well as half of the number. So we have 16 different numbers.

    Explicitly, the sequence is

$$76, 38, 88, 44, 84, 42, 82, 41, 48, 24, 28, 14, 18, 9, 8, 4.$$