

DO NOT TURN TO THE NEXT PAGE until your proctor tells you.

Please read the directions carefully.

- ▶ You have 100 minutes for 32 Problems.
- ▶ Mark your answers on your Answer Form with a pencil.
- ▶ Extra scratch paper is neither given nor allowed. You may use blank pages/spaces in the booklet as scratch paper.
- ▶ There are no penalties for incorrect answers. Answer as many problems as you can; go back and check your work and also go to questions you skip, before the time is over.
- ▶ Calculators are not permitted. Cell phones must be turned off completely and placed out of sight. MathCON problems are ALL done without a calculator.
- ▶ The problems are divided into three categories by difficulty levels:
3 Points (Questions 1-8)
5 Points (Questions 9-24)
7 Points (Questions 25-32)
- ▶ Problems 29-32, the last four problems are constructed-response problems. Enter your numerical answer in the grid on your answer sheet as shown on the right.

1. Although not required, it is suggested that you write your answer from left to right in the boxes at the top of the columns to help you fill in the circles accurately. You will receive credit only if the circles are filled in correctly.

2. Mark no more than one circle in any column.
3. You may start your answers in any column, space permitting. Columns you don't use should be left blanks, and there should be no blank columns between columns that are not blank. For example, if your answer is 201, then either arrangement of filled-in circles shown below is acceptable.

For example: Answer: 201 – either position is correct.

	2	0	1
Ⓐ	Ⓐ	●	Ⓐ
Ⓑ	Ⓑ	Ⓑ	●
Ⓒ	●	Ⓒ	Ⓒ
Ⓓ	Ⓓ	Ⓓ	Ⓓ

2	0	1	
Ⓐ	●	Ⓐ	Ⓐ
Ⓑ	Ⓑ	●	Ⓑ
●	Ⓒ	Ⓒ	Ⓒ
Ⓓ	Ⓓ	Ⓓ	Ⓓ

4. No problem has a negative answer.
- ▶ Notations in Geometry Problems:
- A : Point A
- $\overleftrightarrow{AB}$: Line through points A and B
- $\overline{AB}$: Line segment joining A and B
- AB : Length of the line segment AB
- $\angle ABC$: Angle with the vertex point at B
- $m\angle ABC$: Measure of angle ABC
- $\perp$: Perpendicular
- // : Parallel

1. [Algebra, 3 Points]

A data set contains 12 whole numbers. The mean, median, and mode of the data set are all equal to 6. The smallest number is 5. What is the **largest** possible number of the set, if the mode is unique?

- A) 8 B) 9 C) 10 D) 11 E) 12

3. [Number Theory, 3 Points]

What is the **smallest** whole number n for which $\frac{396}{n}$ is a factor of 240?

- A) 11 B) 22 C) 33 D) 44 E) None of the preceding

4. [Combinatorics, 3 Points]

There are many balls in a lottery sphere. When somebody pushes the button, probability of

- having 1 ball is 50%,
- having 2 balls is 20%, and
- having 3 balls is 30%.

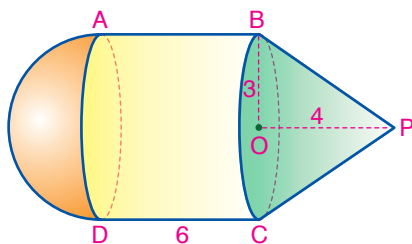
If first Mia and then Paz pushes the button once, what is the probability of obtaining four balls altogether?

- A) 17% B) 19% C) 21% D) 25% E) 34%



6. [Geometry, 3 Points]

In the figure, a hemisphere, a right cylinder, and a right circular cone are glued to each other. The centers of bases of the solids coincide, and each of the radii of three of the solids are the same, where O is the center and P is the vertex (or apex) of the cone.



OP = 4 cm,
OB = 3 cm, and
CD = 6 cm.

What is the volume of the figure?

- A) $84\pi \text{ cm}^3$ B) $82\pi \text{ cm}^3$ C) $80\pi \text{ cm}^3$ D) $78\pi \text{ cm}^3$ E) $72\pi \text{ cm}^3$

7. [Number Theory, 3 Points]

The whole number N is 111 when written in base b, but it is 160 when written in base $b - 2$. What is N in base b - 4?

- A) 323 B) 330 C) 331 D) 332 E) 333

9. [Algebra, 5 Points]

What is the value of $b^2 - 4c$ if the solutions of the quadratic equation $x^2 - bx + c = 0$ are two consecutive integers?

- A) 1 B) 2 C) 3 D) 4 E) Cannot be determined.