



Individual Finals B

1. Let a positive integer n have at least four positive divisors. Let the least four positive divisors be $1 = d_1 < d_2 < d_3 < d_4$. Find, with proof, all solutions to $n^2 = d_1^3 + d_2^3 + d_4^3$.
2. Aumann, Bill, and Charlie each roll a fair 6-sided die with sides labeled 1 through 6 and look at their individual rolls. Each flips a fair coin and, depending on the outcome, looks at the roll of either the player to his right or the person to his left, without anyone else knowing which die he observed. Then, at the same time, each of the three players states the expected value of the sum of the rolls based on the information he has. After hearing what everyone said, the three players again state the expected value of the sum of the rolls based on the information they have. Then, for the third time, after hearing what everyone said, the three players again state the expected value of the sum of the rolls based on the information they have. Prove that Aumann, Bill, and Charlie say the same number the third time.
3. Let ABC be a triangle. Construct three circles k_1, k_2 and k_3 with the same radius such that they intersect each other at a common point O inside the triangle ABC and $k_1 \cap k_2 = \{A, O\}, k_2 \cap k_3 = \{B, O\}, k_3 \cap k_1 = \{C, O\}$. Let t_a be a common tangent of circles k_1 and k_2 such that A is closer to t_a than O . Define t_b and t_c similarly. Those three tangents determine a triangle MNP such that triangle ABC is inside the triangle MNP . Prove that the area of MNP is at least 9 times the area of ABC .