



Algebra A

- [3] On the number line, consider the point x that corresponds to the value 10. Consider 24 distinct integer points $y_1, y_2, \dots, y_{24}$ on the number line such that for all k such that $1 \leq k \leq 12$, we have that y_{2k-1} is the reflection of y_{2k} across x . Find the minimum possible value of

$$\sum_{n=1}^{24} (|y_n - 1| + |y_n + 1|)$$

- [3] Alice, Bob, and Charlie are visiting Princeton and decide to go to the Princeton U-Store to buy some tiger plushies. They each buy at least one plushie at price p . A day later, the U-Store decides to give a discount on plushies and sell them at p' with $0 < p' < p$. Alice, Bob, and Charlie go back to the U-Store and buy some more plushies with each buying at least one again. At the end of that day, Alice has 12 plushies, Bob has 40, and Charlie has 52 but they all spent the same amount of money: \$42. How many plushies did Alice buy on the first day?

- [4] A function f has its domain equal to the set of integers $0, 1, \dots, 11$, and $f(n) \geq 0$ for all such n , and f satisfies

$$f(0) = 0$$

$$f(6) = 1$$

$$\text{If } x \geq 0, y \geq 0, \text{ and } x + y \leq 11, \text{ then } f(x + y) = \frac{f(x) + f(y)}{1 - f(x)f(y)}$$

Find $f(2)^2 + f(10)^2$.

- [4] There is a sequence with $a(2) = 0, a(3) = 1$ and $a(n) = a\left(\left\lfloor \frac{n}{2} \right\rfloor\right) + a\left(\left\lceil \frac{n}{2} \right\rceil\right)$ for $n \geq 4$. Find $a(2014)$. [Note that $\left\lfloor \frac{n}{2} \right\rfloor$ and $\left\lceil \frac{n}{2} \right\rceil$ denote the floor function (largest integer $\leq \frac{n}{2}$) and the ceiling function (smallest integer $\geq \frac{n}{2}$), respectively.]

- [5] Real numbers x, y, z satisfy the following equality:

$$4(x + y + z) = x^2 + y^2 + z^2$$

Let M be the maximum of $xy + yz + zx$, and let m be the minimum of $xy + yz + zx$. Find $M + 10m$.

- [6] Given that $x_{n+2} = \frac{20x_{n+1}}{14x_n}$, $x_0 = 25, x_1 = 11$, it follows that $\sum_{n=0}^{\infty} \frac{x_{3n}}{2^n} = \frac{p}{q}$ for some positive integers p, q with $\text{GCD}(p, q) = 1$. Find $p + q$.

- [7] x, y, z are positive real numbers that satisfy $x^3 + 2y^3 + 6z^3 = 1$. Let k be the maximum possible value of $2x + y + 3z$. Let n be the smallest positive integer such that k^n is an integer. Find the value of $k^n + n$.

- [8] For nonnegative integer n , the following are true:

$$f(0) = 0$$

$$f(1) = 1$$

$$f(n) = f\left(n - \frac{m(m-1)}{2}\right) - f\left(\frac{m(m+1)}{2} - n\right) \text{ for integer } m \text{ satisfying } m \geq 2 \text{ and } \frac{m(m-1)}{2} < n \leq \frac{m(m+1)}{2}.$$

Find the smallest n such that $f(n) = 4$.