PUMaC 2014

Number Theory A

1. [**3**] Let $f(x) = x^3 + ax^2 + bx + c$ have solutions that are distinct negative integers. If $a+b+c = 2014$, find c.

2. [**3**] What is the last digit of $17^{17^{17^{17}}}$?

3. [**4**] Find the number of ending zeros of 2014! in base 9. Give your answer in base 9.

4. [**4**] Find the sum of all positive integer x such that $3 \times 2^x = n^2 - 1$ for some positive integer n.

5. [**5**] Find the number of pairs of integer solution (x, y) that satisfies the equation

$$(x - y + 2)(x - y - 2) = -(x - 2)(y - 2)$$

6. [**6**] Given $S = \{2, 5, 8, 11, 14, 17, 20...\}$. Given that one can choose n different numbers from S, $\{A_1, A_2, ...A_n\}$, s.t. $\sum_{i=1}^{n} \frac{1}{A_i} = 1$. Find the minimum possible value of n.

7. [**7**] Find the number of positive integers $n \leq 2014$ such that there exists integer x that satisfies the condition that $\frac{x+n}{x-n}$ is an odd perfect square.

8. [**8**] Find all number sets (a, b, c, d) s.t. $1 < a \leq b \leq c \leq d$, $a, b, c, d \in \mathbb{N}$, and $a^2 + b + c + d$, $a + b^2 + c + d$, $a + b + c^2 + d$ and $a + b + c + d^2$ are all square numbers. Sum the value of d across all solution set(s).