P U M .·. C

Geometry A

1. Triangle ABC has $AB = BC = 10$ and $CA = 16$. The circle Ω is drawn with diameter BC. Ω meets AC at points C and D. Find the area of triangle ABD.

2. The area of parallelogram $ABCD$ is $51\sqrt{55}$ and $\angle DAC$ is a right angle. If the side lengths of the parallelogram are integers, what is the perimeter of the parallelogram?

3. A right regular hexagonal prism has bases $ABCDEF$, $A'B'C'D'E'F'$ and edges AA', BB', CC', DD', EE', FF', each of which is perpendicular to both hexagons. The height of the prism is 5 and the side length of the hexagons is 6. The plane P passes through points A, C', and E. The area of the portion of P contained in the prism can be expressed as $m\sqrt{n}$, where n is not divisible by the square of any prime. Find $m + n$.

4. An equilateral triangle ABC has side length 7. Point P is in the interior of triangle ABC, such that $PB = 3$ and $PC = 5$. The distance between the circumcenters of ABC and PBC can be expressed as $\frac{m\sqrt{n}}{p}$, where n not divisible by the square of any prime and m and p are relatively prime positive integers. What is $m + n + p$?

5. Rectangle $HOMF$ has $HO = 11$ and $OM = 5$. Triangle ABC has orthocenter H and circumcenter O. M is the midpoint of BC and altitude AF meets BC at F. Find the length of BC.

6. Triangle ABC has $\angle A = 90°$, $AB = 2$, and $AC = 4$. Circle ω_1 has center C and radius CA, while circle ω_2 has center B and radius BA. The two circles intersect at point E, different from point A. Point M is on ω_2 and in the interior of ABC, such that BM is parallel to EC. Suppose EM intersects ω_1 at point K and AM intersects ω_1 at point Z. What is the area of quadrilateral $ZEBK$?

7. Let $ACDB$ be a cyclic quadrilateral with circumcircle ω. Let $AC = 5, CD = 6$, and $DB = 7$. Suppose that there is a unique point P on ω such that $\overline{PC}$ intersects $\overline{AB}$ at a point P_1 and $\overline{PD}$ intersects $\overline{AB}$ at a point P_2, such that $AP_1 = 3$ and $P_2B = 4$. Let Q be the unique point on ω such that $\overline{QC}$ intersects $\overline{AB}$ at a point Q_1, $\overline{QD}$ intersects $\overline{AB}$ at a point Q_2, Q_1 is closer to B than P_1 is to B, and $P_2Q_2 = 2$. The length of P_1Q_1 can be written as $\frac{p}{q}$ where p and q are relatively prime positive integers. Find $p + q$.

8. Triangle ABC with $AB = 4, BC = 5, CA = 6$ has circumcircle Ω and incircle ω. Let Γ be the circle tangent to Ω and the sides AB, BC, and let $X = \Gamma \cap \Omega$. Let Y, Z be distinct points on Ω such that XY, XZ are tangent to ω. Find YZ^2.

   The following fact may be useful: if $\triangle ABC$ has incircle ω with center I and radius r, and $\triangle DEF$ is the intouch triangle (i.e. D, E, F are intersections of the incircle with BC, CA, AB, respectively) and H is the orthocenter of $\triangle DEF$, then the inversion of X about ω (i.e. the point X' on ray IX such that $IX' \cdot IX = r^2$) is the midpoint of DH.