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Geometry B

1. Equilateral triangle ABC has area 1. A', B' and C' are the midpoints of BC, CA and AB, respectively. A'', B'' and C'' are the midpoints of $B'C'$, $C'A'$ and $A'B'$, respectively. The area of trapezoid $BB''C''C$ can be written as $\frac{m}{n}$ for relatively prime positive integers m and n. Find $m + n$.

2. A kite is inscribed in a circle with center O and radius 60. The diagonals of the kite meet at a point P, and OP is an integer. The minimum possible area of the kite can be expressed in the form $a\sqrt{b}$, where a and b are positive integers and b is squarefree. Find $a + b$.

3. Triangle ABC has $AB = BC = 10$ and $CA = 16$. The circle Ω is drawn with diameter BC. Ω meets AC at points C and D. Find the area of triangle ABD.

4. The area of parallelogram $ABCD$ is $51\sqrt{55}$ and $\angle DAC$ is a right angle. If the side lengths of the parallelogram are integers, what is the perimeter of the parallelogram?

5. A right regular hexagonal prism has bases $ABCDEF$, $A'B'C'D'E'F'$ and edges AA', BB', CC', DD', EE', FF', each of which is perpendicular to both hexagons. The height of the prism is 5 and the side length of the hexagons is 6. The plane P passes through points A, C', and E. The area of the portion of P contained in the prism can be expressed as $m\sqrt{n}$, where n is not divisible by the square of any prime. Find $m + n$.

6. An equilateral triangle ABC has side length 7. Point P is in the interior of triangle ABC, such that $PB = 3$ and $PC = 5$. The distance between the circumcenters of ABC and PBC can be expressed as $\frac{m\sqrt{n}}{p}$, where n not divisible by the square of any prime and m and p are relatively prime positive integers. What is $m + n + p$?

7. Rectangle $HOMF$ has $HO = 11$ and $OM = 5$. Triangle ABC has orthocenter H and circumcenter O. M is the midpoint of BC and altitude AF meets BC at F. Find the length of BC.

8. Triangle ABC has $\angle A = 90°$, $AB = 2$, and $AC = 4$. Circle ω_1 has center C and radius CA, while circle ω_2 has center B and radius BA. The two circles intersect at point E, different from point A. Point M is on ω_2 and in the interior of ABC, such that BM is parallel to EC. Suppose EM intersects ω_1 at point K and AM intersects ω_1 at point Z. What is the area of quadrilateral $ZEBK$?