

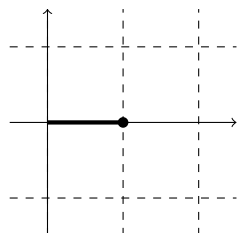


Algebra B

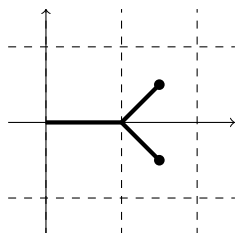
1. Let $A = \sqrt{7 + 2\sqrt{10}} - \sqrt{7 - 2\sqrt{10}}$. We can express A as $a\sqrt{b}$, where a, b are integers and b is square-free. Compute $a + b$.
2. Alien Tanvi has a favorite number, but somehow she's managed to forget it. She remembers that it can be written as $x^2 + \frac{1}{x^2}$, where x is a real number satisfying $x^4 + 4x^2 + \frac{4}{x^2} + \frac{1}{x^4} = 523$. What is Alien Tanvi's favorite number?
3. Consider polynomial $f(x) = ax^3 + bx^2 + cx + d$ where a, b, c, d are nonnegative integers satisfying $ab + bc + cd + ad = 20$. Find the sum of all distinct possible values of $f(1)$.
4. Let f and g be two polynomials such that $f(g(x)) = g(f(x))$. If $g(x)$ is linear but not identically equal to x , and $f(x) = x^3 + 60x^2 + 1000x + c$ for some c , find the value of c .

Note: This problem was scrapped from the contest. It should state: "... If $g(x)$ is linear, **non-constant**, and not identically equal to x , ..."

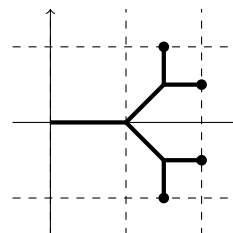
5. T_1 consists of a single branch from 0 to 1 in the complex plane. This branch then splits into two branches at the endpoint which each form a 135° angle with the branch in T_1 , and each branch has length $\frac{1}{\sqrt{2}}$. This process is repeated, so that from each terminal branch in T_n , we form two more branches at angles 135° with $\frac{1}{\sqrt{2}}$ the length. Let L_n be the collection of the 2^{n-1} endpoints of the tree T_n . If multiple terminal branches end at the same point, then that point is counted multiple times in L_n . Shown below is T_k for $k = 1, 2, 3$ with dots at the points in L_k . Find the sum of ℓ^2 over all points ℓ in L_{10} .



(a) T_1 and L_1



(b) T_2 and L_2



(c) T_3 and L_3

6. Compute the number of solutions to $1 + \cos(\theta) + \cos(2\theta) + \cos(3\theta) + \dots + \cos(2024\theta) = \frac{1}{2}$ for $\theta \in [0, 2\pi]$.
7. Real numbers a, b, c satisfy $\frac{1}{ab} = b + 2c$, $\frac{1}{bc} = 2c + 3a$, $\frac{1}{ca} = 3a + b$. Then $(a + b + c)^3$ can be written as $\frac{m}{n}$ for relatively prime positive integers m and n . Find $m + n$.
8. Let $\{a_n\}_{n=0}^\infty$ be the sequence defined by the recurrence relation $a_{n+3} = 2a_{n+2} - 23a_{n+1} + 3a_n$ for all $n \geq 0$, with initial conditions $a_0 = 20$, $a_1 = 0$, and $a_2 = 23$. Let $b_n = a_n^3$ for all $n \geq 0$. Then there exists a unique positive integer k and constants $c_0, \dots, c_{k-1}$ with $c_0 \neq 0$ and $c_{k-1} \neq 0$ such that for all sufficiently large n , we have the recurrence relation $b_{n+k} = \sum_{t=0}^{k-1} c_t b_{n+t}$. Find $k + \sqrt{|c_{k-1}|} + \sqrt{|c_0|}$.



Name:

Team:

Write answers in table below:

Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8