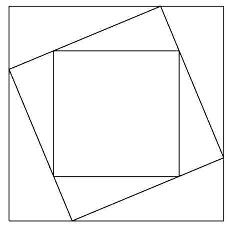
P U M ∴ C

Geometry A

1. The following three squares are inscribed within each other such that they all share the same center, and the largest and smallest squares have parallel sides. If the largest square has side length 17 and the middle square has side length 13, the side length of the smallest square can be expressed in the form $\frac{a}{b}$, where a and b are relatively prime positive integers. Find $a+b$.



2. Let $ABCD$ be a square of side length 1 made of paper. Let M be the midpoint of side AB. Let E be a point on side BC and let F be a point on side AD such that A lands on line ME when $\triangle AMF$ is folded across line MF and such that B lands on line MF when $\triangle BME$ is folded across line ME. The area of the resulting shape when both folds are made can be written as $\frac{a-\sqrt{b}}{c}$ for positive integers a, b, c with b squarefree. Find $a+b+c$.

3. Let $\triangle ABC$ be a right triangle with $\angle A = 90°$ and $AB = 1$. Let x be the length that AC must be so that the perpendicular bisector of AC is tangent to the incircle of $\triangle ABC$. Let y be the length that BC must be so that the perpendicular bisector of BC is tangent to the incircle of $\triangle ABC$. (Note that x and y arise in different triangles.) Then $x+y=\frac{m}{n}$ for positive integers m, n with $\frac{m}{n}$ in simplest form. Compute $m+n$.

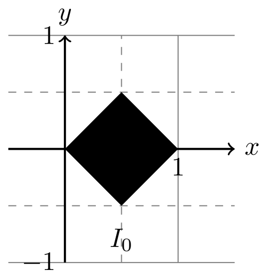
4. Let $\triangle ABC$ be such that $AB = 15, BC = 13, CA = 14$. Let P be the point on the circumcircle of $\triangle ABC$ such that P is on the same side of $\overline{AB}$ as C and $AP = BP$. Let X be the foot of the perpendicular from P to AC. Then the length of AX is $\frac{m}{n}$ for some relatively prime positive integers m and n. Find $m+n$.

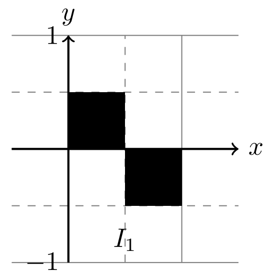
5. Let $\triangle ABC$ be a triangle such that the angle bisector of $\angle BAC$, the median from B to side AC, and the perpendicular bisector of AB intersect at a single point X. If $AX = 5$ and $AC = 12$, compute BC^2.

   Note: The problem should conclude: BC^2 can be expressed in the form $\frac{m}{n}$, where m and n are relatively prime positive integers. Compute $m+n$.

6. Let $\triangle ABC$ be a triangle with $AB = 10$. Let D be a point on the opposite side of line AC as B so that $\triangle ACD$ is directly similar to $\triangle ABC$ (i.e. $\angle ACD = \angle ABC$, etc). Let M be the midpoint of AD. Given that A is the centroid of triangle $\triangle BCM$, compute BC^2.

7. The following is the construction of the *twindragon* fractal. I_0 is the solid square region with vertices at $(0,0), (\frac{1}{2},\frac{1}{2}), (1,0), (\frac{1}{2},-\frac{1}{2})$. Recursively, the region I_{n+1} consists of two copies of I_n: one copy which is rotated 45° counterclockwise around the origin and scaled by a factor of $\frac{1}{\sqrt{2}}$, and another copy which is also rotated 45° counterclockwise around the origin and scaled by a factor of $\frac{1}{\sqrt{2}}$ and then translated by $(\frac{1}{2},-\frac{1}{2})$. We have displayed I_0 and I_1 below. Let I_∞ be the limiting region of $I_0, I_1, \ldots$. The area of the smallest convex polygon which encloses I_∞ can be written as $\frac{a}{b}$ for relatively prime positive integers a and b. Find $a+b$.





8. Let E be the ellipse lying in the x, y plane centered at $(0, 0)$ with semi-major axis of length 2 along the x-axis and semi-minor axis of length 1 along the y-axis. Let C be a cone created by revolving two perpendicular lines about an angle bisector of the perpendicular angle. There are some points (x, y, z) where the vertex of C could be so that E is the intersection of C with the x, y plane. These points define a convex polygon in the x, z plane. The area of this polygon can be expressed as $\sqrt{n}$ for some positive integer n. Find n.

   (Some definitions: the semi-major axis is the longest distance from the center of the ellipse to the boundary, and the semi-minor axis is the shortest distance from the center of the ellipse to the boundary.)

Name:

Team:

Write answers in table below:

Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8