P U M ∴ C

Individual Finals A

1. Given $x, y, z \geq 0$ and $x\sqrt{yz} + y\sqrt{xz} + z\sqrt{xy} \geq 12$, find, with proof, the minimum value of $x + y + z$.

2. Moby has an infinite piece of paper, an infinite straightedge, and a magic triangle. The paper starts with two points a distance 1 away. With the straightedge, he can connect any two points to form a line. With the magic triangle, he can take two points A and B and draw the two points C and D such that $\triangle ABC$ and $\triangle ABD$ are equilateral triangles. He can also draw a point at the intersection of any two lines or line segments. With finitely many operations, can Moby draw a square on the paper? A proof in the affirmative should consist of a clear depiction or description of steps taken.

3. There are 2024 perfectly logical assassins in a circle, named 1 to 2024 in clockwise order. Turns move clockwise around the circle among currently alive assassins, starting with assassin 1 and cycling back to the front while more than one assassin is alive. On each assassin's turn, they must kill someone else (they are not allowed to pass on their turn). Each assassin has a bounty associated with them, with k dollars on assassin k. At a given assassin's turn, their top priority is to be the last man standing, and the second priority is to kill the assassin that has the highest bounty currently alive (other than themself). Who is the last man standing?