



Individual Finals B

1. Let n be a positive integer. Find, with proof, the smallest integer k for which every subset of $\{1, 2, \dots, 2024n\}$ of size k contains two elements a, b such that $\gcd(a, b) \leq 2023$.
2. Given $x, y, z \geq 0$ and $x\sqrt{yz} + y\sqrt{xz} + z\sqrt{xy} \geq 12$, find, with proof, the minimum possible value of $x + y + z$.
3. Moby has an infinite piece of paper, an infinite straightedge, and a magic triangle. The paper starts with two points a distance 1 away. With the straightedge, he can connect any two points to form a line. With the magic triangle, he can take two points A and B and draw the two points C and D such that $\triangle ABC$ and $\triangle ABD$ are equilateral triangles. He can also draw a point at the intersection of any two lines or line segments. With finitely many operations, can Moby draw a square on the paper? A proof in the affirmative should consist of a clear depiction or description of steps taken.