P U M ∴ C

2025 Algebra A

You have **50 minutes** for this examination.
Do not discuss or distribute these problems until **January 16th, 2026**.

1. Find the sum of all real values of x satisfying

$$\sqrt{x+\sqrt{x+\sqrt{x+\cdots}}} = x - \sqrt{x-\sqrt{x-\sqrt{x-\cdots}}}.$$

2. Let $\star$ be an associative operation on the set of positive real numbers that satisfies

$$\underbrace{x \star x \star \cdots \star x}_{x \text{ appears } k \text{ times}} = \frac{x}{\sqrt{k}}$$

for any positive real number x and positive integer k. Let s_n denote the nth smallest positive integer with no prime factor greater than 5. Compute $s_1 \star s_2 \star s_3 \star \cdots$.

3. Find the largest possible value of

$$\frac{-2x+6y}{1+2x^2+3y^2}$$

over all real numbers x, y.

4. Let

$$S = \sum_{j=1}^{2025} \binom{4050}{2025+j} \cos(3j).$$

What is the largest positive integer n such that 2^n divides $\lfloor S \rfloor$?

5. Let $f(x) = 2x^2 - 1$. Find the smallest positive integer N such that the median of the real roots of the polynomial $\underbrace{(f \circ f \circ f \circ \ldots f \circ f)}_{f \text{ composed } N \text{ times}}(x) - x$ has absolute value at most 2^{-2025}.

6. Let c be a real number, and let $f(t) = (t - 0.01) + (t^2 - c)i$ for every real number t. Suppose that $m(t)$ is a (not necessarily continuous) function from $\mathbb{R}$ to $\mathbb{C}$ such that

$$m(t)^2 + 1 + \frac{1}{m(t)^2} = (t-1)(t+1)$$

for every real number t. Over all such functions $f(t)$ and $m(t)$, what is the maximum possible number of distinct ordered pairs of real numbers (a, b) such that $f(a) = m(b)$?

7. Let

$$\omega = e^{i\pi/2025} = \cos\left(\frac{\pi}{2025}\right) + i\sin\left(\frac{\pi}{2025}\right).$$

David writes the complex numbers $2, \omega, \omega^2, \ldots, \omega^{4049}$ in a line going from left to right on a blackboard. Then, each minute, if the two leftmost complex numbers on the blackboard are z_1 and z_2, David erases the two numbers and replaces them with

$$\frac{1}{z_1 + z_2} - \frac{1}{\frac{1}{z_1} + \frac{1}{z_2}}.$$

After 4049 minutes, there is only one number remaining. What is the last remaining number?

8. Let S be the set of ordered triples of positive integers (x, y, z) such that $y(x+y+z) = zx$ and

$$x^2 + y^2 + z^2 + xy + yz + zx = 2\frac{(x+y)(y+z)(z+x)}{x+y+z} + 145.$$

Find the sum of $100x + 10y + z$ over all $(x, y, z) \in S$.