



2025 Algebra B

You have **50 minutes** for this examination.

Do not discuss or distribute these problems until **January 16th, 2026**.

1. There exist real numbers m and n , as well as a quadratic polynomial f , such that for all real y , we have $f(x) = m(y + n)^2$ if and only if $5x^2 + 3y^2 + 4x + 12y = 0$. Compute n .
2. Max writes an integer on the board. Then every minute, if the number k on the board is even, he replaces it with $\frac{k}{2}$; otherwise, he replaces it with $\frac{3k+1}{2}$. If Max writes down the number 1 after at most 5 minutes, how many different integers could he have started with?
3. Find the sum of all real values of x satisfying

$$\sqrt{x + \sqrt{x + \sqrt{x + \cdots}}} = x - \sqrt{x - \sqrt{x - \sqrt{x - \cdots}}}.$$

4. Let $\star$ be an associative operation on the set of positive real numbers that satisfies

$$\underbrace{x \star x \star \cdots \star x}_{x \text{ appears } k \text{ times}} = \frac{x}{\sqrt{k}}$$

for any positive real number x and positive integer k . Let s_n denote the n th smallest positive integer with no prime factor greater than 5. Compute $s_1 \star s_2 \star s_3 \star \cdots$.

5. Find the largest possible value of

$$\frac{-2x + 6y}{1 + 2x^2 + 3y^2}$$

over all real numbers x, y .

6. Let

$$S = \sum_{j=1}^{2025} \binom{4050}{2025+j} \cos(3j).$$

What is the largest positive integer n such that 2^n divides $\lfloor S \rfloor$?

7. Let $f(x) = 2x^2 - 1$. Find the smallest positive integer N such that the median of the real roots of the polynomial $\underbrace{(f \circ f \circ f \circ \cdots \circ f \circ f)}_{f \text{ composed } N \text{ times}}(x) - x$ has absolute value at most 2^{-2025} .

8. Let c be a real number, and let $f(t) = (t - 0.01) + (t^2 - c)i$ for every real number t . Suppose that $m(t)$ is a (not necessarily continuous) function from $\mathbb{R}$ to $\mathbb{C}$ such that

$$m(t)^2 + 1 + \frac{1}{m(t)^2} = (t - 1)(t + 1)$$

for every real number t . Over all such functions $f(t)$ and $m(t)$, what is the maximum possible number of distinct ordered pairs of real numbers (a, b) such that $f(a) = m(b)$?