P U M :. C

2025 Geometry A

You have **50 minutes** for this examination.
Do not discuss or distribute these problems until **January 16th, 2026**.

1. Two circles ω_1 and ω_2 have radii 3 and 2, respectively, and intersect at two distinct points X and Y. The line through X perpendicular to $\overline{XY}$ intersects ω_1 at $A \neq X$ and ω_2 at $B \neq X$. Given that $AB = 4$, find the area of triangle ABY.

2. Consider a right triangular prism with bases $\triangle ABC$ and $\triangle A'B'C'$, where $\triangle ABC \cong \triangle A'B'C'$ and $AC = 10$. Point M lies on $\overline{AB}$ such that $AM = 5$ and $MB = 4$. Let N lie on $\overline{A'C'}$ such that $\overline{BN}$ is parallel to the plane of $\triangle A'CM$. Find $A'N$.

3. Let $ABCD$ be an isosceles trapezoid with $\overline{AB} \parallel \overline{CD}$, $AB = 20$, and $CD = 22$. Let M and N be the midpoints of sides AD and BC, respectively. Ray NM meets the circumcircle of $ABCD$ at K. If the lengths of KM and AM are positive integers, find the sum of all possible values of KM.

4. Let ABC be a triangle with $\angle BAC = 45^\circ$, $AB = 2\sqrt{2}$ and $AC = 3$. Let X be the reflection of B over $\overline{AC}$ and let Y be the reflection of C over $\overline{AB}$. Lines XY and BC intersect at S. Let H be the orthocenter of $\triangle ABC$ and let O be the circumcenter of $\triangle XHY$. Find SO.

5. Let ABC be a triangle with $\angle BAC = 60^\circ$, $AB = 10$, $AC = 13$, and orthocenter H. Let ω_1 be the circle through H tangent to $\overline{BC}$ at B, and let ω_2 be the circle through H tangent to $\overline{BC}$ at C. Let $\overline{CH}$ intersect ω_1 again at U and let $\overline{BH}$ intersect ω_2 again at V. Let N be the point on the same side of $\overline{BC}$ as A such that triangle BNC is equilateral. Find $\frac{NU}{NV}$.

6. Let ABC be a triangle with $AB = 7$, $BC = 6$ and $AC = 8$. Let M be the midpoint of $\overline{BC}$ and let S be on segment AM such that $\frac{MS}{AM} = \frac{1}{4}$. The line through A parallel to $\overline{BC}$ meets the circumcircles of triangles ASB and ASC at $D \neq A$ and $E \neq A$, respectively. Find DE.

   Remark. Regrettably, due to an error on behalf of the author, the in-contest problem statement had the ratio written as $\frac{MS}{AM} = 4$ instead of $\frac{MS}{AM} = \frac{1}{4}$.

7. Let ABC be a triangle with $AC = 33$, $BC = 16$, $AB = 28$ and incenter I. Let U and V be points on sides AB and AC respectively such that $AU = AV = 20$. Let P be the reflection of B over U and let Q be the reflection of C over V. The circumcircles of triangles BIP and CIQ intersect at a point $S \neq I$. Let M be the midpoint of $\overline{BC}$. Find MS.

8. Connor is standing on an infinite, flat ground plane at $z = 0$. The ground is perfectly solid everywhere except for a circular hole of radius 1 centered at the origin, i.e., the set $x^2 + y^2 \leq 1$ is missing from the plane. He holds a cube of side length 1000 starting entirely above the ground in the half-space $z > 0$. He may translate and rotate the cube in any way he likes, subject to the constraint that no part of the cube ever passes through the solid portion of the ground plane; it may only pass through the circular hole.

   Let S be the set of all points with $z < 0$ that can coincide with a vertex of the cube during such motions. Find the volume of S.