PUMaC

2025 Geometry B

You have **50 minutes** for this examination.
Do not discuss or distribute these problems until **January 16th, 2026**.

1. Let ABC be a triangle with area 21 and $BC = 7$. Square $XYZT$ is such that X lies on segment AB, Y lies on segment AC, and Z and T lie on segment BC. Find the side length of square $XYZT$.

2. Let ABC be an equilateral triangle with side length 1. Let D be the midpoint of $\overline{BC}$, and let E and F be points on $\overline{CA}$ and $\overline{AB}$, respectively, such that $AE = BF$. Given that the area of $\triangle DEF$ is $\frac{1}{3}$ the area of $\triangle ABC$, find the smallest possible value of AE.

3. Two circles ω_1 and ω_2 have radii 3 and 2, respectively, and intersect at two distinct points X and Y. The line through X perpendicular to $\overline{XY}$ intersects ω_1 at $A \neq X$ and ω_2 at $B \neq X$. Given that $AB = 4$, find the area of triangle ABY.

4. Consider a right triangular prism with bases $\triangle ABC$ and $\triangle A'B'C'$, where $\triangle ABC \cong \triangle A'B'C'$ and $AC = 10$. Point M lies on $\overline{AB}$ such that $AM = 5$ and $MB = 4$. Let N lie on $\overline{A'C'}$ such that $\overline{BN}$ is parallel to the plane of $\triangle A'CM$. Find $A'N$.

5. Let $ABCD$ be an isosceles trapezoid with $\overline{AB} \parallel \overline{CD}$, $AB = 20$, and $CD = 22$. Let M and N be the midpoints of sides AD and BC, respectively. Ray NM meets the circumcircle of $ABCD$ at K. If the lengths of KM and AM are positive integers, find the sum of all possible values of KM.

6. Let ABC be a triangle with $\angle BAC = 45°$, $AB = 2\sqrt{2}$ and $AC = 3$. Let X be the reflection of B over $\overline{AC}$ and let Y be the reflection of C over $\overline{AB}$. Lines XY and BC intersect at S. Let H be the orthocenter of $\triangle ABC$ and let O be the circumcenter of $\triangle XHY$. Find SO.

7. Let ABC be a triangle with $\angle BAC = 60°$, $AB = 10$, $AC = 13$, and orthocenter H. Let ω_1 be the circle through H tangent to $\overline{BC}$ at B, and let ω_2 be the circle through H tangent to $\overline{BC}$ at C. Let $\overline{CH}$ intersect ω_1 again at U and let $\overline{BH}$ intersect ω_2 again at V. Let N be the point on the same side of $\overline{BC}$ as A such that triangle BNC is equilateral. Find $\frac{NU}{NV}$.

8. Let ABC be a triangle with $AB = 7$, $BC = 6$ and $AC = 8$. Let M be the midpoint of $\overline{BC}$ and let S be on segment AM such that $\frac{MS}{AM} = \frac{1}{4}$. The line through A parallel to $\overline{BC}$ meets the circumcircles of triangles ASB and ASC at $D \neq A$ and $E \neq A$, respectively. Find DE.

   Remark. Regrettably, due to an error on behalf of the author, the in-contest problem statement had the ratio written as $\frac{MS}{AM} = 4$ instead of $\frac{MS}{AM} = \frac{1}{4}$.