HMMT November 2025

November 08, 2025

Team Round

1. **[20]** Compute the number of ways to divide an 8×8 square into 3 rectangles, each with (positive) integer side lengths.

2. **[25]** Mark writes the squares of several distinct positive integers (in base 10) on a blackboard. Given that each nonzero digit appears exactly once on the blackboard, compute the smallest possible sum of the numbers on the blackboard.

3. **[30]** Let $ABCD$ and $CEFG$ be squares such that C lies on segment $\overline{DG}$ and E lies on segment $\overline{BC}$. Let O be the circumcenter of triangle AEG. Given that A, D, and O are collinear and $AB = 1$, compute FG.

4. **[35]** For positive integers n and k with $k > 1$, let $s_k(n)$ denote the sum of the digits of n when written in base k. (For instance, $s_3(2025) = 5$ because $2025 = 2210000_3$.) A positive integer n is a *digiroot* if $s_2(n) = \sqrt{s_4(n)}$. Compute the sum of all digiroots less than 1000.

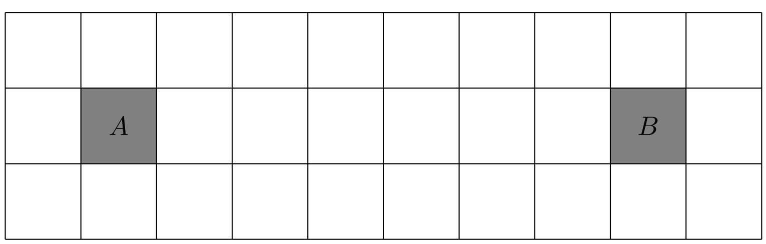
5. **[40]** Kelvin the frog is in the bottom-left cell of a 6×6 grid, and he wants to reach the top-right cell. He can take steps either up one cell or right one cell. However, there is a raccoon in one of the 36 cells uniformly at random, and Kelvin's path must avoid this raccoon. Compute the expected number of distinct paths Kelvin can take to reach the top-right cell.

   (If the raccoon is in either the bottom-left or top-right cell, then there are 0 such paths.)

6. **[45]** Let P be a point inside triangle ABC such that $BP = PC$ and $\angle ABP + \angle ACP = 90^\circ$. Given that $AB = 12$, $AC = 16$, and $AP = 11$, compute the area of the concave quadrilateral $ABPC$.

7. **[45]** Let S be the set of all positive integers less than 143 that are relatively prime to 143. Compute the number of ordered triples (a, b, c) of elements of S such that $a + b = c$.

8. **[50]** Alexandrimitrov is walking in the 3×10 grid below. He can walk from a cell to any cell that shares an edge with it. Given that he starts in cell A, compute the number of ways he can walk to cell B such that he visits every cell exactly once. (Starting in cell A counts as visiting cell A.)



9. **[55]** Let a, b, and c be positive real numbers such that

$$\sqrt{a} + \sqrt{b} + \sqrt{c} = 7,$$
$$\sqrt{a+1} + \sqrt{b+1} + \sqrt{c+1} = 8,$$
$$(\sqrt{a+1} + \sqrt{a})(\sqrt{b+1} + \sqrt{b})(\sqrt{c+1} + \sqrt{c}) = 60.$$

   Compute $a + b + c$.

10. **[55]** Let $ABCD$ be an isosceles trapezoid with $\overline{AB}$ parallel to $\overline{CD}$, and let P be a point in the interior of $ABCD$ such that

$$\angle PBA = 3\angle PAB \quad \text{and} \quad \angle PCD = 3\angle PDC.$$

   Given that $BP = 8$, $CP = 9$, and $\cos\angle APD = \frac{2}{3}$, compute $\cos\angle PAB$.