ID: ______________

1. __________

2. __________

3. __________

4. __________

5. __________

6. __________

7. __________

8. __________

9. __________

10. __________

11. __________

12. __________

– 75 minutes
– no calculators
– positive integer answers

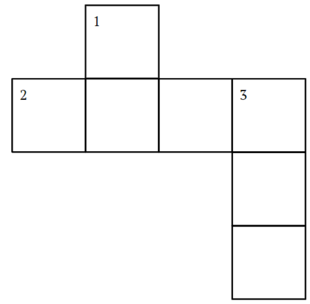
Girls in Math

Name: ________________________ **Team:** ________________________

2024 Individual Round

1. Trains run from New York to New Haven, departing every half-hour beginning at 6:00 AM. Trains run from New Haven to Boston, departing every 45 minutes beginning at 6:15 AM. The distance between New Haven and New York is 75 miles and the distance between New Haven and Boston is 200 miles. Trains can travel at 50 miles per hour. Jessica decides to depart from New York at 6:00 AM. What is the shortest amount of time it would take her to get to Boston by train, in minutes, if she must pass through New Haven?

2. Cat and Claire are playing a crossword game. Cat says: "I know the digits of 2-Across are all the same, and 3-Down is a perfect square whose digits are in strictly ascending order." Claire, who previously only knew the number in 1-Down, says "The number in 1-Down is divisible by 13, and with your information I now know how to complete the grid." What is the number in 1-Down?



3. Five 1s and four 0s are distributed in a 3×3 grid at random. The probability there exists a row, column, or diagonal with all three numbers being the same can be expressed as $\frac{m}{n}$ where m, n are relatively prime positive integers. Find $m + n$.

4. Ten equally spaced points $A, B, C, D, E, F, G, H, I, J$ are drawn on the circumference of a unit circle. What is $AB^2 + AC^2 + AD^2 + \ldots + AJ^2$?

5. What is $\displaystyle\sum_{k=1}^{12} \sqrt{(k-1)k(k+1)(k+2)+1}$?

6. A *subsequence* of a word is formed by taking some characters of the word in order. For example, ATTIC is a subsequence of the word $\text{M}\underline{\text{AT}}\text{HEMA}\underline{\text{TIC}}\text{S}$, formed by the underlined characters. Compute the number of distinct 4 character subsequences of the word MATHEMATICS.

7. In acute triangle ABC, M is the midpoint of BC, and point D located on segment AC. Let BD and AM intersect at E. Let CE and AB intersect at F. Call the line parallel to BC passing through A line ℓ. Then, let ℓ intersect BE at G and ℓ intersect CE at H, and we have $[BCF] = [CHD] + [HDG]$. The value of $\frac{[AFD]}{[BEC]}$ when written in simplest form is $\frac{m}{n}$. Find $m + n$.

8. Find the number of ordered tuples $(k_1, \ldots, k_n)$ satisfying $\displaystyle\sum_{i=1}^{n} \frac{1}{k_i} = 1$ and $\displaystyle\prod_{i=1}^{n} k_i \leq 2024$.

9. In triangle ABC, let D be the point on segment BC such that $\angle BAD = \angle ACB$. The circle through A tangent to BC at D intersects AB at $E \neq A$ and AC at $F \neq A$. Let lines EF and BC intersect at a point P. If $AE = 11$, $DE = 10$, and $EF = 20$, compute the perimeter of triangle PDF.

10. Find the greatest positive integer n such that $(n+1)!$ divides

$$\left(13231^n - 1\right)\left(13231^{n-1} - 1\right) \ldots \left(13231 - 1\right).$$

11. For integers $k > 1$ define $f(k)$ as the sum of all numbers of the form $\frac{1}{n}$ such that $\frac{1}{n}$ terminates when written in base k. For example, in base 10, $\frac{1}{2} = 0.5$ terminates but $\frac{1}{3} = 0.333...$ does not terminate, so $\frac{1}{2}$ would be part of the sum representing $f(10)$ but $\frac{1}{3}$ would not. If $\displaystyle\sum_{k \mid 10!, k>1} f(k)$ can be expressed as $\frac{m}{n}$ such that m, n are relatively prime positive integers, find $m + n$.

12. Find the minimum possible value of $n - m$, where n, m are real numbers such that $m \leq \frac{4+b^2}{b} + \frac{4+a^2}{a} \leq n$ for all positive real numbers a, b that satisfy $\frac{a^2+1}{a} + \frac{b^2+1}{b} \leq 18$ and $a^2b + b^2a \leq 3$.