



Girls in Math 2024 Tiebreaker Round

1. In triangle ABC , $AB = 20$ and $AC = 23$. Let M be the midpoint of BC , and let A' be the reflection of A over M . Let D and E be the feet of the perpendiculars from A' to AB and AC , respectively. The circumcircle of $\triangle MDE$ meets BC again at a point $X \neq M$. If $MX = 3$, the length BC can be represented as $\frac{m}{n}$, where m, n are relatively prime positive integers. Find $m + n$.

1. _____
Answer



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2. Collatz has a function

$$f(x) = \begin{cases} x/2 & \text{if } x \text{ is even} \\ 5x + 1 & \text{if } x \text{ is odd} \end{cases}$$

Define $f_n(x)$ where $f_1(x) = f(x)$ and $f_n(x) = f(f_{n-1}(x))$ when $n \geq 2$. What is $f_{2024} \left(\sum_{k=0}^{30} 2^{4k+1} + 2^{4k} \right)$?

2. _____
Answer



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3. Jacqueline has three (not necessarily distinct) polynomials $p(x)$, $q(x)$, and $r(x)$ such that each polynomial is one of $x + 1$, $x + 2$, or $x + 3$. For each combination of $p(x)$, $q(x)$, and $r(x)$, Jacqueline finds the product $p(x)q(x)r(x)$. Jacqueline then sums all of the products (which are, again, not necessarily distinct) to obtain the polynomial $w(x)$. Find the sum of the coefficients of $w(x)$.

3. _____
Answer