Girls in Math 2026 Guts Round Solutions

Yale Math Competitions

February 2026

1. Compute $\frac{(2!+0!+2^6)+\frac{201}{5}}{(2!+0!+2^6)-\frac{201}{5}}$.

   Proposed by: Dieter Yang

   Answer: $\boxed{4}$

   Solution: Noting that $(2!+0!+2^6) = 2+1+64 = 67$ and $201 = 67 \cdot 3$, we can divide both sides by 67.
$$\frac{(2!+0!+2^6)+\frac{201}{5}}{(2!+0!+2^6)-\frac{201}{5}} = \frac{1+\frac{3}{5}}{1-\frac{3}{5}} = \frac{\frac{8}{5}}{\frac{2}{5}} = \boxed{4}$$

2. Square $ABCD$ has side length 6, and point P is such that $AP = BP = 5$. If the greatest possible length of segment $\overline{CP}$ can be expressed as $\sqrt{a}$, what is the value of a?

   Proposed by: Dieter Yang

   Answer: $\boxed{109}$

   Solution: Set up coordinates with $A = (0,0)$, $B = (0,6)$, $C = (6,6)$, and $D = (6,0)$. Since $AP = BP$, we have that P has y-coordinate 3 (perpendicular bisector), so $3 = (x,3)$ where $x^2 + 3^2 = 5^2$, meaning $x = \pm 4$. The maximum value of PC will occur when $x = -4$, so $PC^2 = 10^2 + 3^2 = 109$ and our answer is $\boxed{109}$.

3. What is the minimum number of chess pieces (rooks or bishops) that must be placed on an 8×8 chessboard so that each rook attacks exactly one bishop and each bishop attacks exactly one rook? Rooks can attack along a row or column, and bishops can attack along a diagonal.

   Proposed by: Aron Fekete

   Answer: $\boxed{4}$

   Solution: A rook and a bishop cannot both simultaneously attack each other. If there are either zero rooks or bishops then the other type cannot be attacking it, so this is impossible. If there is just one of either type, then it must be attacked by all pieces of the other type meaning it cannot be attacking any of them, so this is impossible. Therefore, there must be at least two rooks and at least two bishops. This is possible by placing rooks on $(2, 1)$ and $(1, 2)$ and bishops on $(3, 2)$ and $(2, 3)$. Therefore, our minimum is $\boxed{4}$.

4. The answer to this problem can be expressed as $2025 \cdot \frac{p}{q}$ where p and q are positive, relatively prime integers. Given that $p + q = 67$, what is pq?

   Proposed by: Dieter Yang

   Answer: $\boxed{990}$

   Solution: Since the answer to this question is equal to pq, and the answer is also equal to $2025 \cdot \frac{p}{q}$, we can set these equal and get

$$pq = 2025 \cdot \frac{p}{q} \implies q = \frac{2025}{q} \implies q^2 = 2025 \implies q = 45$$

   since q is positive. Since $p + q = 67 \implies p + 45 = 67$, we get that $p = 22$, we have that the answer is $pq = 45 \cdot 22 = \boxed{990}$.

5. Brianna has nine solid cubes with side lengths $1, 2, \ldots, 9$. She paints each large cube only on its outer surface, then cuts each into $1 \times 1 \times 1$ cubes. Brianna selects a single unit cube uniformly at random. Let the probability that the chosen unit cube has all six faces unpainted be $\frac{m}{n}$, where m and n are relatively prime positive integers. Find $m + n$.

   Proposed by: Grant Zhang

   Answer: $\boxed{2809}$

   Solution: A cube with a side length of $n \geq 2$ will have $(n-2)^3$ possible cubes with no paint, since there is an inner cube with a side length of two less (reduced by one from each side). Therefore our sum is (we will use a well-known identity here that $1^3 + \cdots + n^3 = (1 + \cdots + n)^2$) the following:

$$\frac{1^3 + 2^3 + \cdots + 7^3}{1^3 + 2^3 + \cdots + 9^3} = \frac{(1 + 2 + \cdots + 7)^2}{(1 + 2 + \cdots + 9)^2} = \frac{(\frac{7 \cdot 8}{2})^2}{(\frac{9 \cdot 10}{2})^2} = \frac{28^2}{45^2} = \frac{784}{2025}$$

   so the answer is $784 + 2025 = \boxed{2809}$

6. Traffic in Queen Beatrice's beehive is controlled by stop signs. The regular octagon-shaped signs are painted yellow along the borders. 50 stop signs need to be painted. If the side length of the stop sign is 1.5 mm, and the side length of the inner edge of the border is 1.2 mm, and each bucket of paint can cover 50 mm^2, how many full buckets of paint are required?

   Proposed by: Lily Jiang

   Answer: $\boxed{4}$

   Solution: The area of a regular octagon with side length s is well-known to be $2s^2(1 + \sqrt{2})$.

Then the side length of a complete stop sign is 1.5 millimeters, with their inner octagon having side length 1.2. Thus the area of the white border for any given octagon is

$$2 \cdot 1.5^2 \cdot (1+\sqrt{2}) - 2 \cdot 1.2^2 \cdot (1+\sqrt{2}) = 2(1+\sqrt{2}) \cdot (0.81) = 1.62(1+\sqrt{2}).$$

As we have 50 stop signs total, we see that the total area across all octagons is then $50 \cdot 1.62(1+\sqrt{2}) = 81(1+\sqrt{2})$. Then using $\sqrt{2} \approx 1.414$, we can compute $81(1+\sqrt{2}) = 195.55$, so we see that $\boxed{4}$ buckets (to fill 200 mm^2) suffices.

7. $ABCDEF$ is a regular hexagon, with center O. A bee starts at A, and every second it moves to an adjacent vertex or O (or if it is at O, it can go to any other vertex). How many ways are there for it to be at D in exactly 4 seconds?

   Proposed by: Henry Burton

   Answer: $\boxed{18}$

   Solution: Let X denote an unknown letter that is not equal to O. The sequences of 4 letters the bee moves to can be in the form $\underline{OXOD}$, $\underline{XOXD}$, $\underline{XXXD}$, $\underline{XXOD}$, and $\underline{OXXD}$. For the first category, X can be any of the six vertices. For the second category, there are two choices for the first X and two choices for the second X, leading to four total. For the third form, there are zero solutions by parity, since the bee must end on either vertex A, C, E. For the forth form, there are 4 possible decisions for the first two moves, leading to four total sequences. For the last type, there are two ways to choose the second to last character, and then two ways for each to choose the previous move, leading to four total solutions. Therefore, our answer is $4+4+4+6 = \boxed{18}$.

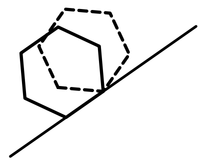
8. We call a month `perfect` if the month starts on a Sunday and ends on a Saturday. How many perfect months are there from January 1st, 2026 up until December 31st, 2126? A year that is divisible by 400, or divisible by 4 but not 100 is a leap year. Today is Saturday, February 28th, 2026.

   Proposed by: Dieter Yang

   Answer: $\boxed{12}$

   Solution: Observe that only February can be a perfect month. In addition, we need February to have 28 days in order for this to happen so we have to avoid leap years. Since there are 365 days in year, which is 1 mod 7, then we know that the days will shift over by 1. In addition, we know that leap years will shift by 2. However, in 2032, we have that this is a leap year so we have to wait 5 more years (there is an extra shift from 2032 leap year and the 2036 leap year). This means that February 2037 has the next perfect month. After that, we have February 2043 since there is only 1 leap year in between the years. Then we have two leap years in the next 6 years so February 2049 is no longer a perfect month. Continuing this process, we can conclude that February 2054, February 2065, February 2071, February 2082, February 2093, February 2099, February 2105, February 2111, and February 2122. Therefore our answer is $\boxed{12}$.

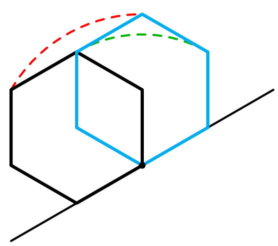
9. Sisyphus the Bee, has been sentenced to an eternity of punishment, where he must roll a honeycomb (which is a regular hexagon of side length 1) up a slope. The area that the hexagon sweeps after one roll can be written in the form $\frac{a\sqrt{b}+c\pi}{d}$, where a, c, and d are relatively prime and b is square free. Find $1000a+100b+10c+d$. Note: Rolling the hexagon involves rotating the hexagon into the air while keeping one pivot vertex fixed on the ground at all times.



Proposed by: Benjamin Wu

Answer: $\boxed{9346}$

Solution: Below is a diagram of the area swept out by the honeycomb. Essentially, we take each vertex of the hexagon and rotate it along the pivot point. The only vertex that matters is the one diametrically opposite of the vertex, the rest all sweep an area that is contained inside the hexagon (e.g., the green dotted line). Now, we can compute the area: we can split the swept area into three portions: a $60°$ arc (dotted-red below) with a half-hexagon on both the left and right. These half-hexagons add up to an entire hexagon, which has area $\frac{3\sqrt{3}}{2}$, and since the arc has radius 2, it has area $\frac{1}{6}(4\pi) = \frac{2}{3}\pi$. Combining gives $\frac{9\sqrt{3}+4\pi}{6}$, so the answer is $\boxed{9346}$.



10. Bonnie the Bee starts on a hexagonal cell in the middle of an infinite honeycomb. Each second, Bonnie moves to the middle of a neighboring hexagonal cell. How many different cells could Bonnie be on after following this procedure for one minute?

Proposed by: Grant Zhang

Answer: $\boxed{10981}$

Solution: One can draw the honeycomb in rings in terms of the minimum distance Ben is from the starting honeycomb. Each subsequent ring has 6 more cells than its predecessor (i.e. there are 6 more cells that have a minimum distance of $x+1$ from the starting cell than the number of cells that have a minimum distance of x from the center); this is most easily figured out by drawing a diagram. Then, notice that if Ben moves for $x \geq 2$

seconds, he can end up on any of the cells that have a minimum distance of x from the starting cell since each cell has 6 neighbors. Therefore, after moving for a minute, Ben can visit $6(1)+6(2)+6(3)+\ldots+6(60)+1 = \boxed{10981}$ cells, with the final 1 being the starting cell itself.

11. Bridget the queen bee orders Sleepy the worker bee to review n problems and write m problems, where n, m are non-negative integers and $n+m = 100$. The ith problem reviewed takes $2i$ energy and the jth problem written takes $3j$ energy. What is the minimum energy Sleepy can use?

    Proposed by: Dieter Yang

    Answer: $\boxed{6120}$

    Solution: Observe that the energy it costs to review n problems is precisely

$$\sum_{i=1}^{n} 2i = n(n+1)$$

and the energy it costs to write m problems is precisely

$$\sum_{j=1}^{m} 3j = 1.5m(m+1) = 1.5(100-n)(101-n)$$

Combining and simplifying, we get that the total energy is determined by

$$n(n+1) + 1.5(100-n)(101-n) = 2.5n^2 - 300.5n + 15150$$

and recall that the minimum occurs at the vertex of this parabola which is given by $n = 60.1$. Trying both $n = 60, 61$, we get that the minimum energy occurs at $n = 60$ with $\boxed{6120}$ energy.

12. Point P is inside square $ABCD$. Given that $\angle DAP = \angle ADP = 15^\circ$, find $\angle CPD$, in degrees.

    Proposed by: Jacqueline Lu

    Answer: $\boxed{75}$

    Solution: Construct point G in triangle CDG such that $\angle CDG = \angle GDC = 15^\circ$. The diagram becomes symmetric across diagonal $\overline{BD}$, so we can see that $\triangle DPG$ is an isosceles triangle. $\angle GDP = 90^\circ - \angle ADP - \angle CDG = 60^\circ$, and thus $\angle DPG = \angle DGP = \frac{180^\circ - \angle GDP}{2} = 60^\circ$. Thus, $\triangle DPG$ is an equilateral triangle, meaning $GP = GD = GC$. We now seek to find $\angle GPC$, as $\angle CPD = \angle DPG + \angle GPC$. $\triangle GPC$ is an isosceles triangle, and $\angle CGP = 360^\circ - \angle DGC - \angle DGP = 150^\circ$. Hence, $\angle GPC = \frac{180^\circ - \angle CGP}{2} = 15^\circ$. Finally, $\angle CPD = 60^\circ + 15^\circ = \boxed{75^\circ}$.

13. We say that two points $(a, b), (c, d)$ are `equivalent` when both $a - c$ and $b - d$ are integers. What is the minimum number of points $P = \{p_1, p_2, \ldots, p_n\}$ in the plane needed such that

any point (x, y) is at most a distance $\frac{1}{2}$ away from a point equivalent to some p_i in P?

Proposed by: Noah Ripke

Answer: $\boxed{2}$

Solution: Observe that any point (a, b) in the plane is equivalent to a point in $[0, 1] \times [0, 1]$ by just taking the fractional part of a, b. Thus, it suffices to cover the square $[0, 1] \times [0, 1]$ by circles of radius $\frac{1}{2}$. A covering exists where $p_1 = (\frac{1}{2}, \frac{1}{2})$ and $p_2 = (0, 0)$. Every point within $\frac{1}{2}$ of the center of $[0, 1] \times [0, 1]$ is close to p_1, and every point near the corners is within $\frac{1}{2}$ of one of the corners $(0, 0), (1, 0), (0, 1), (1, 1)$ all of which are equivalent to p_2. Now, suppose a covering of one point exists. Observe that the area of a circle of radius $\frac{1}{2}$ is $\frac{1}{4}\pi$. But $\pi < 4$ so the area of the circle is less than 1. But the area of the square is $1 \cdot 1 = 1$. So a single circle cannot cover the entire square. Thus $\boxed{2}$ is the minimum number of points.

14. A deck of cards labeled 1 through 100 is dealt. What is the length of the largest set of cards such that no card is directly divisible by any other in the set, unless that quotient is a power of 2 or a power of 5? For example, 9 and 6 can both be dealt in the same sequence, as well as 36 and 9, but not 9 and 3.

Proposed by: Brady Exoo

Answer: $\boxed{67}$

Solution: $\{34, 35, ..., 100\}$ is one construction. To show a better solution is impossible, partition $\{1, 2, ..., 100\}$ into sets based on what they become when divided by their largest factor that is a power of 3 (i.e. $\{1, 3, 9, 27, 81\}, \{2, 6, 18, 54\}, \{4, 12, 36\}, ...$). There are 67 such sets (one for each number from 1 to 100 that isn't divisible by 3), and only one number in each set can be in the final set, making $\boxed{67}$ an upper bound.

15. Consider a sequence that alternates between being arithmetic and geometric. Let $a_1 = 1$. Define for $n \geq 2$, $a_n = a_{n-1} + 2$ if n even and $a_n = 3a_{n-1}$ if n odd. If $a_{2027} - a_{2026} = a \cdot 3^b - 2$, with a and b positive integers and a not divisible by 3, what is $a + b$?

Proposed by: Madhav Krishnaswamy

Answer: $\boxed{1020}$

Solution: Start with a general form: Let $a_1 = 1$, the common difference be d, and the common ratio be r. Then $a_2 = 1 + d$, $a_4 = r(1 + d) + d$, $a_6 = r^2(1 + d) + rd + d$, $\ldots$, $a_{2k} = r^{k-1}(1 + d) + d(r^{k-1} + r^{k-2} + \ldots + 1)$, with k a positive integer. Using the geometric series sum formula $a_{2k} = r^{k-1} + d\frac{r^k - 1}{r-1}$, and since $a_{2k+1} = r \cdot a_{2k}$, $a_{2k+1} = r^k + d\frac{r^{k+1} - r}{r-1}$. Plugging in $d = 2, r = 3$, we get $a_{2k} = 3^k + 3^{k-1} - 1$ and $a_{2k+1} = 3^{k+1} + 3^k - 3$. Therefore $a_{2k+1} - a_{2k} = 3^{k+1} - 3^{k-1} - 2 = 8 \cdot 3^{k-1} - 2$. Since we want $a_{2027} - a_{2026}$, $k = 1013$, so the answer is $8 + 1012 = \boxed{1020}$.

16. How many functions from $\{1, 2, \ldots, 8\}$ to $\{1, 2, 3\}$ are there such that $f(x)f(x + 1)f(x + 2)$

is even for all $1 \leq x \leq 6$?

Proposed by: Owen Zhang

Answer: $\boxed{1825}$

Solution: Essentially, we are looking for the number of sequences a_n such that every three consecutive numbers contains at least one 2. Define x_n, y_n, z_n to be the number of such sequences of length n where the last term is 2, where the last term is odd and the second-to-last term is 2, and where the last two terms are both odd, respectively.

Then, we can construct the following recurrences: $x_{n+1} = x_n + y_n + z_n$, since we can always append a 2. $y_{n+1} = 2x_{n+1}$, since the second-to-last element must be a 2, and we can either append 1 or a 3, and $z_{n+1} = 2y_n$, since the last element must be odd, and since we have 2 choices of an odd element to append, the third-to-last element must be 2 in order for the sequence to be valid.

Then, our initial conditions by inspection are $x_1 = 1, y_1 = 2, z_1 = 0$. We can calculate $x_8 = 739, y_8 = 594, z_8 = 492$, for a total answer of $\boxed{1825}$.

17. How many ways are there to color a 5×5 grid with white or black squares subject to the following conditions:
(1) All squares along the diagonal from the bottom left corner to top right corner are black.
(2) The grid is symmetric about the diagonal from the bottom left corner to top right corner.
(3) If a black square in the ith row and a black square in the jth row appear in the same column, then the square in the ith row and jth column is black. (Rows are indexed from bottom to top, and columns indexed from left to right.)

Proposed by: Henry Greenwold

Answer: $\boxed{52}$

Solution: Let (i, j) denote the square in the ith row and jth column. We give a bijection between the set of partitions of $\{1, 2, 3, 4, 5\}$ and the set of possible colorings as follows. Given a partition of $\{1, 2, 3, 4, 5\}$, we color (i, j) black if and only if i and j lie in the same subset in the partition. It's easy to verify that this satisfies the given conditions for a coloring. Injectivity follows since for any two distinct partitions, there must exist i and j such that i and j lie in the same subset in one partition and different subsets in the other. These give different colorings for (i, j). Surjectivity is immediate too since given a valid coloring, we can construct a corresponding partition of $\{1, 2, 3, 4, 5\}$ by placing i and j in the same subset if and only if (i, j) is black. This is well defined by the conditions of a coloring.

Thus it suffices to find the number of valid partitions of $\{1, 2, 3, 4, 5\}$. We can group these by the size of each subset. One subset of size 5 gives 1 partition. Subsets of sizes $4, 1$ gives 5 partitions. Subsets of sizes $3, 2$ gives $\binom{5}{2} = 10$ partitions. Subsets of sizes $3, 1, 1$ gives $\binom{5}{2} = 10$ partitions. Subsets of sizes $2, 2, 1$ gives $5 \cdot 3 = 15$ partitions. Subsets of sizes $2, 1, 1, 1$ gives $\binom{5}{2} = 10$ partitions. Subsets of sizes $1, 1, 1, 1, 1$ gives $\binom{5}{2} = 1$ partition. Thus there are $1 + 5 + 10 + 10 + 15 + 10 + 1 = \boxed{52}$ partitions in total.

18. There are 6 distinct lines in a plane, and no three intersect at one point. There are n points where two lines intersect. Find the sum of all possible values of n.

Proposed by: Henry Burton

Answer: $\boxed{87}$

Solution:

In a plane, any two distinct lines either intersect at exactly one point or are parallel. The problem specifies that no three lines intersect at one point, which means every intersection is formed by exactly two lines.

The maximum number of intersection points occurs when no two lines are parallel, simply being

$$\binom{6}{2} = \frac{6 \cdot 5}{2} = 15.$$

If lines are parallel, the number of intersections decreases. We categorize the lines into sets of parallel lines. Let $m_1, m_2, \dots, m_k$ be the sizes of these parallel sets, such that $m_1 + m_2 + \cdots + m_k = 6$. Then the count of intersection points $\binom{6}{2}$ decreases by $\binom{m_i}{2}$ for each i, so the total number of intersection points n is calculated by:

$$n = 15 - \sum_{i=1}^{k} \binom{m_i}{2}.$$

We can then see that the different values of n are given by different solutions to $m_1 + \cdots + m_k = 6$, i.e., the different partitions of 6. We enumerate them all below:

Partition (m_i)	$15 - \sum \binom{m_i}{2}$	n
$\{6\}$	$15 - 15$	**0**
$\{5, 1\}$	$15 - 10$	**5**
$\{4, 2\}$	$15 - (6 + 1)$	**8**
$\{4, 1, 1\}$	$15 - 6$	**9**
$\{3, 3\}$	$15 - (3 + 3)$	**9**
$\{3, 2, 1\}$	$15 - (3 + 1)$	**11**
$\{3, 1, 1, 1\}$	$15 - 3$	**12**
$\{2, 2, 2\}$	$15 - (1 + 1 + 1)$	**12**
$\{2, 2, 1, 1\}$	$15 - (1 + 1)$	**13**
$\{2, 1, 1, 1, 1\}$	$15 - 1$	**14**
$\{1, 1, 1, 1, 1, 1\}$	$15 - 0$	**15**

Thus, the set of distinct possible values for n is $S = \{0, 5, 8, 9, 11, 12, 13, 14, 15\}$, and hence

$$0 + 5 + 8 + 9 + 11 + 12 + 13 + 14 + 15 = \boxed{87}$$

is the sum of all possible values of n.

19. For how many nonnegative integers n does $2^n + 3^n$ divide $4^n + 9^n$?

Proposed by: Patrick Lu

Answer: $\boxed{1}$

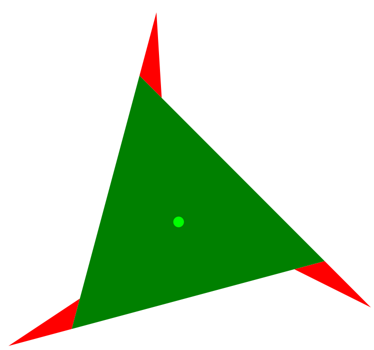
Solution: We have
$$\frac{4^n + 9^n}{2^n + 3^n} = 3^n - 2^n + \frac{2^{2n+1}}{2^n + 3^n},$$
so $2^n + 3^n$ must be a power of 2. Note that if $n \geq 1$, $2^n + 3^n$ is odd and greater than 1, so only $n = 0$ works and there is $\boxed{1}$ solution.

20. It is possible to create a polygon such that there exists a point in the polygon such that standing at that point one can't see the entirety of any side. (We say a point P cannot `see` the entirety of an edge E if there exists a point $P' \in E$ where P' is not an endpoint of E and the segment $\overline{PP'}$ intersects some edge other than E.) What is the minimum number of edges for such a polygon?

Proposed by: Brady Exoo

Answer: $\boxed{6}$

Solution: Observe that if you are inside a triangle, it is always possible to see all edges of the triangle. In a polygon with 5 or fewer sides, you can split the polygon into triangles, and every triangle always shares an edge with the polygon. Therefore, any such polygon must have at least 6 sides, and indeed, a configuration with $\boxed{6}$ sides is shown below:



21. Consider triangle $\triangle ABC$ with circumcircle Ω and incenter I. Let M be the arc midpoint of major arc BC and N the midpoint of minor arc AB. Lines MN and AB meet at X, and then line XI and side BC intersect at K. Given that $AB = 7$, $AC = 10$, and $BC = 12$, it can be found that $AK = \frac{\sqrt{m}}{n}$ for positive integers m, n with n minimized. Find m.

Proposed by: Yusuf Sheikh

Answer: $\boxed{154}$

Solution: The key realization is that K is the midpoint of BC. Proof: Say K' is the midpoint of BC. Let P be the midpoint of minor arc BC, then Pascal's Theorem on hexagon $NBPCMA$ implies that $NM \cap BA$, $NC \cap AP$, $BC \cap PM$ are collinear. But then $X = NM \cap BA$, $I = NC \cap AP$, and $BC \cap PM$ is clearly K' by symmetry. But K' is the intersection of BC and XI as X, I, K' are collinear, and thus $K = K'$ as desired. Then by Stewart's Theorem, (or Apollonius) we find

$$AK = \frac{\sqrt{2b^2 + 2c^2 - a^2}}{2} = \frac{2 \cdot 7^2 + 2 \cdot 10^2 - 12^2}{2} = \frac{\sqrt{154}}{2},$$

giving us $\boxed{154}$.

22. (22-24) This set contains six problems, which can be grouped into three pairs. The two questions in each pair have the same answer. You will receive $5N^2$ points, where N is the number of pairs in which you give the correct answer for both problems. No other points will be awarded.

Question A: Consider a unfair coin; initially, the probability of flipping heads and tails are both $\frac{1}{2}$. However, for each flip following the first flip, the probability that the result is the same as the previous flip is $\frac{1}{3}$. In expectation, how many flips will we need to perform before we observe a consecutive sequence of HHH or TTT?

Question B: Let a number be `superprime` if the sum of its digits, product of its digits, and all possible permutations of it are prime. The answer to this question is a superprime, and moreover, if you reverse the digits of the answer to this question, you get the largest *superprime* that exists.

Question C: Beth is a very busy bee, so every day she only has the brain capacity to think of a single number. Today, she is thinking of some number N such that the congruence $x^2 \equiv 1 \pmod{N}$ has exactly 32 solutions modulo N, N is divisible by 9, N has exactly 60 positive divisors, and N is not divisible by 5, 11, or 13. Moreover, since Beth is very busy, N is the smallest positive integer that satisfies these properties. What is N?

Question D: How many ordered triplets of integers (a, b, c) are there such that $-10 \leq a, b, c \leq 10$ and $ab + c^2 = 0$?

Question E: A security hub has 6 identical sensors placed at the vertices of a regular hexagon. Each sensor can be either `Armed` or `Disarmed`. Two configurations are considered the same if one can be transformed into the other by any symmetry of the hexagon (i.e., rotations or reflections). How many different configurations are there?

Question F: Consider 5 points such that no three are collinear. Draw an edge between every pair. How many ways are there to color these edges with red, blue, and yellow such that there is no triangle using the points as vertices with three edges of the same color?

Proposed by: Owen Zhang

Answer: $\boxed{13, 113, 17136, 113, 13, 17136}$

Solution:

- (a) Let E_i be the expected flips needed to terminate if our current run of identical flips has length i. Trivially $E_3 = 0$. After 1 flip, we are in state E_1. The next flip will match with probability $\frac{1}{3}$, so $E_1 = 1 + \frac{1}{3}E_2 + \frac{2}{3}E_1$. From E_2, we get $E_2 = 1 + \frac{1}{3}E_3 + \frac{2}{3}E_1 = 1 + \frac{2}{3}E_1$. Solving gives $E_1 = 12$, so the total number of flips is just $1 + 12 = \boxed{13}$, accounting for the initial flip.

- (b) If the product of the digits is prime, then all digits except one are prime, and the non-1 digit is either 2, 3, 5, or 7. However, we cannot have it be 2 or 5, since then a permutation like $1 \cdots 12$ or $1 \cdots 15$ would be composite.

  Observe that our number is $10^n + 10^{n-1} + \cdots + 1 + c \cdot 10^k$ for $c = 2$ or 6 for some $k \leq n$. However, observe that if $n \geq 5$, then as k varies from 0 to n (under permutations), $c \cdot 10^k$ will sweep all nonzero residues mod 7, so it will eventually make the number divisible by 7. Therefore, we only need to check numbers that have 5 or fewer digits.

  For two-digit numbers, we can have $13, 31$ or $17, 71$. For three-digit numbers, we have $113, 131, 311$ (7 does not work since the digit sum makes every three-digit candidate divisible by 3). For four-digit numbers, there are no possibilities (3 does not work since the digit sum is 9, and 7 does not work since 1711 is composite). For five-digit numbers, there are no possibilities, since 11711 is composite and 13111 is composite. Therefore, the larget superprime is 311, and its reverse is $\boxed{113}$.

- (c) Observe that $x^2 \equiv 1 \pmod{N}$ is only true if $x^2 \equiv 1 \pmod{p^e}$ for every prime power dividing N by the Chinese Remainder Theorem. In particular, $x^2 \equiv 1 \pmod{p^e}$ has only two solutions: ± 1 for $p > 2$, and $x^2 \equiv 1 \pmod{2}$ has 1 solution, $x^2 \equiv 1 \pmod{4}$ has 2 solutions, and $x^2 \equiv 1 \pmod{2^k}$ for $k \geq 3$ has 4 solutions. Since the total number of solutions is 32, this is the product of the number of solutions in each prime power. Since $9|N$, then 9 contributes 2 to the product. If N was odd, then 4 other prime factors are required. However, since N has 60 divisors, the maximum number of distinct prime factors is 4 (since $60 = 2 \cdot 2 \cdot 3 \cdot 5$), so N is even. We also must have that $4|N$, since otherwise we still will have 5 distinct prime factors. Suppose 8 does not divide N. Then, the number of factors of N is $3 \cdot 3 \cdot C$, but 9 doesn't divide 60. Therefore, $8|N$, so we can write $N = 2^a 3^b p_1^{e_1} p_2^{e_2}$, with $a \geq 3, b \geq 2$. We must have $e_1 = e_2 = 1$, since otherwise the number of divisors of N would exceed 60. Therefore, the only possible solution is $2^4 3^2$, and $7, 17$ are the smallest remaining primes, which gives $\boxed{17136}$.

- (d) If $c = 0$, then either $a = 0$ or $b = 0$, so the number of valid ordered pairs is $21 + 21 - 1 = 41$ (subtracting 1 for overcounting the $(0, 0)$ case). For $c \neq 0$, a, b must have opposite signs. We can solve $ab = c^2$ for positive a, b and multiply by 2 to apply the choice of sign. Checking $c = 1$ through 10 gives a total of $2, 6, 6, 6, 2, 6, 2, 2, 2, 2$ solutions, for a total of 36. Doubling for the sign gives 72. The total is therefore $41 + 72 = \boxed{113}$.

- (e) If there are 0 or 1 armed sensors, there is only one configuration. If there are 2 armed sensors, there are 3 configurations (either they are adjacent, apart by 1, or opposite). If there are 3 armed sensors, there are 3 configurations (they can be consecutive, or there can be two adjacent with one apart by one, or they can all be spaced apart by 1). The 4, 5, 6 cases are symmetric, so the total is $1+1+3+3+3+1+1 = \boxed{13}$.
- (f) This is the hardest subproblem, so the intended approach was to solve the other 5 and therefore realize that this question must be paired with (c) to get $\boxed{17136}$. For the sake of space, the solution is omitted, but the intended approach for solving directly would begin by caseworking on the number of colors on the edges of vertex 1 (there are 4 cases: $(4,0,0); (3,1,0); (2,2,0); (2,1,1)$).

25. Let p denote the number of individuals who scored above a 5 and a denote the number of teams that scored above a 5 at GIM 2026, and q denote the number of individuals who scored above a 5 and b denote the number of teams that scored above a 5 at last year's (2025) GIM. Compute $ap + qb$. If the correct answer is A and you provide an estimate E, you will recieve

$$\max\left(0, \left\lfloor 20 - \sqrt{\frac{|E-A|}{2}} \right\rfloor\right)$$

points.

Proposed by: Dieter Yang

Answer: $\boxed{623}$

26. An antichain of $\{1, 2, \ldots, n\}$ is a set of subsets $A_i \subseteq \{1, 2, \ldots n\}$, where $A_i \subsetneq A_j$. if $i \neq j$. What is the number of antichains of $\{1, 2, 3, 4, 5, 6\}$? If the correct answer is A and you give an estimate E, you will recieve

$$\max\left(0, \left\lfloor 20 \cdot \left(1 - \ln\left(\max\left(\left|\frac{A}{E}\right|, \left|\frac{E}{A}\right|\right)\right)\right) + 0.5 \right\rfloor\right)$$

points.

Proposed by: Dieter Yang

Answer: $\boxed{7828354}$

Solution: This number is the value of $n = 6$ in the Dedekind number sequence $M(n)$. $M(n)$ is only known up to $n = 9$, and the values ≥ 9 are an open problem. $M(6) = \boxed{7828354}$. $M(n)$ can be roughly approximated by the number of antichains where each subset has length $\lfloor \frac{n}{2} \rfloor$ or $2^{\binom{n}{\lfloor n/2 \rfloor}}$. For $n = 6$, $M(n) \geq 2^{\binom{6}{3}} = 2^{20}$. This gets you within the right order of magnitude of $M(6)$.

27. $ABCDEF$ is a regular hexagon, with center O. A bee starts at A, and every second it moves to an adjacent vertex or O (or if it is at O, it can go to any other vertex). How many digits are in the total number of ways for the bee to be at D in exactly 2026 seconds? If the correct answer is A, and you give an estimate E, you will receive

$$\max\left(0, \left\lfloor 20 - \frac{|A-E|}{20} \right\rfloor\right)$$

points.

Proposed by: Benjamin Wu

Answer: $\boxed{1138}$

Solution: The following code gets the answer via dynamic programming:

```
arr = [1, 0, 0, 0, 0, 0, 0]
b = [0, 0, 0, 0, 0, 0, 0]
iters = 2026
for _ in range(iters):
    b[6] = sum(arr[:6])
    for i in range(6):
        b[i] = arr[(i+1)%6] + arr[(i+5)%6] + arr[6]
    for i in range(7):
        arr[i] = b[i]
print(arr[3])
```

To achieve a good estimate, observe that after a large number of paths, they will eventually converge to a relatively even distribution where the center is 6 times as likely as the edges which will roughly have $\frac{1}{12}$ of the paths. Therefore a good approximation is given by roughly $3.5^{2026} \cdot \frac{1}{12}$ (there are 3 paths to take on the perimeter and 6 in the middle) which has about 1102 digits. (The correct answer is approximately 1.74×10^{1137})