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- 75 minutes
- No calculators
- Positive integer answers
- Write answers INSIDE the box

NAME

TEAM

Girls in Math 2026  
Individual Round

1. For any real numbers  $a, b$ , define  $a \triangleright b = 2a - b$ . Evaluate  $(311 \triangleright 131) + (131 \triangleright 113) + (113 \triangleright 311)$ .
2. Parallelogram  $ABCD$  has area 128.  $E, F, G, H$  and  $I$  are the midpoints of  $\overline{AB}, \overline{BE}, \overline{CD}, \overline{FG}$ , and  $\overline{CF}$  respectively. What is the area of quadrilateral  $HICG$ ?
3. Brenda has tasked you with buying the necessary materials to build the wire frame of a table. Each inch of wire costs you \$4. The wire will be used for the four legs of the table, as well as the four edges of the tabletop. The volume contained by the tabletop will be a rectangular prism, and Brenda wants this volume to be at least  $500 \text{ in}^3$ . What's the minimum price required to buy the necessary materials?
4. Find the sum of all real values of  $x$  that satisfy the equation  $\log_3(x + 5) - 2\log_9(x - 1) = 1$ .
5. For a finite set of integers  $A$ , we define the difference set  $A - A := \{a_i - a_j \mid a_i, a_j \in A\}$  to be the set of all differences of two (possibly not distinct) elements of  $A$ . Let  $N$  be the minimum of  $|A - A|$  over all subsets  $A \subset \mathbb{Z}$  with  $|A| = 99$ , and let  $M$  be the maximum of  $|A - A|$  over all subsets  $A \subset \mathbb{Z}$  with  $|A| = 99$ . Find  $N + M$ .
6.  $ABCD$  is a square with side length 40. Points  $E, F$  are chosen on  $\overline{AB}, \overline{CD}$ , respectively, such that  $AE = 1$  and  $CF = 9$ . Point  $X_1$  is chosen on  $\overline{AB}$ , points  $X_2$  and  $X_3$  are chosen on  $\overline{BC}$ , and point  $X_4$  is chosen on  $\overline{CD}$  so that  $\angle EX_iF = 90^\circ$  for  $i = 1, 2, 3, 4$ . Given that the polygon  $X_1X_2X_3X_4$  is non-degenerate, find its area.
7. Let  $N$  be the number of functions  $f$  from the integers  $\{1, 2, \dots, 25\}$  to integers in  $\{1, 2, \dots, 25\}$  such that  $f(i) \neq f(j)$  for all  $i \neq j$  and  $f(i) + j \equiv i + f(j) \pmod{5}$  for all  $1 \leq i < j \leq 25$ . How many divisors does  $N$  have?
8. For a positive integer  $n$ , let  $\Omega(n)$  denote the number of prime factors of  $n$ , counted with multiplicity. What is the smallest positive integer  $n$  for which  $2^{\Omega(n)} < n < 2 \cdot 2^{\Omega(n)}$ ?
9. Let  $S$  be the set of prime numbers less than 2026. For  $p \neq q$ ,  $p$  and  $q$  are considered **connected** if  $pq \equiv k \pmod{2026}$ , for some integer  $k$  with  $0 < k < 2026$ . The degree of  $p$  is the number of  $q \in S$  such that  $p$  and  $q$  are connected. For what value of  $k$  is the maximum degree of an element in  $S$  maximized?
10. Triangle  $ABC$  is inscribed in a circle centered at  $O$  with radius 5. It is given that  $BC = 6$  and the three distances from  $O$  to the sides of  $ABC$  form the side lengths of a right triangle. The area of  $ABC$  can be expressed as  $\frac{m}{n}$ , where  $m$  and  $n$  are relatively prime positive integers. What is  $m + n$ ?
11. Define a **move** to be an operation on a binary string where we select four consecutive digits 0110 in the string and replace it with 1001. What is the maximal number moves can we perform to a length 100 binary string before no further moves are possible, across all  $2^{100}$  starting strings?
12. Twenty points are evenly spaced on a unit circle. Three distinct points  $A, B$ , and  $C$  are chosen from them uniformly at random. The expected value of  $AB^2 + BC^2 + CA^2$  can be expressed in the form  $\frac{m}{n}$ , where  $m$  and  $n$  are relatively prime positive integers. What is  $m + n$ ?

