



TEAM NAME

1. If a , b , and c are positive real numbers satisfying $(a+b)(b+c) = 9$, $(b+c)(c+a) = 12$, $(c+a)(a+b) = 27$, then $a+b+c$ can be expressed as $\frac{m}{n}$, where m and n are relatively prime positive integers. What is $m+n$?

2. Compute $\sum_{n=1}^{2026} \left\lfloor \log_2 \left\lceil \frac{n}{2} \right\rceil \right\rfloor$.

3. Beatrice writes the sequence $2, 1, 3, 4, \dots$, where each term after the second term is the sum of the previous two terms. Find the remainder when the product of the first 2026 numbers is divided by 20.

4. In an 8-person tournament, there are 4 first-round matches, each with a skilled player vs. a unskilled player. Skilled players have a $\frac{3}{4}$ chance of beating unskilled players. Players of the same type (unskilled vs. unskilled, skilled vs. skilled) have a $\frac{1}{2}$ chance of beating each other. The winner of each round will play the winner of the other game on their side of the bracket in Round 2. Round 3 is the match between the final two players. The probability that the winner of the tournament is unskilled is $\frac{m}{n}$, where m, n are relatively prime positive integers. Find $m+n$.

5. Let ABC be a triangle with circumcircle Ω . Let D and E be the feet of the altitudes from B and C , respectively. Extend $\overline{DE}$ to intersect Ω at distinct points K and L . Let M be the midpoint of $\overline{KL}$. If $\angle B = 40^\circ$ and $\angle C = 70^\circ$, what is the measure of $\angle BAM$, in degrees?

6. Define a sequence of polynomials by $F_1(x) = (x-1)(x-2)\cdots(x-2025)$, $F_2(x) = (x-1^2)(x-2^2)\cdots(x-2025^2)$, and for $n \geq 3$, $F_n = F_{n-1}^2 + F_1^2$ if n is odd and $F_n = F_{n-1}^2 F_2^2$ if n is even. How many distinct real roots does F_{2025} have?

7. Cat and Claire are discussing Claire's favorite two-digit number.

Cat: "If you told me the smaller of the two digits of your number and the number of factors of your number, but not which one was which, would I know your number?"

Claire: "No, but if I told you which one was which, you would!"

Cat: "What if you instead told me the larger of the two digits of your number and the number of factors of your number, but not which one was which?"

Claire: "You still wouldn't know, but if I told you which one was which you would!"

Cat: "Aha! Now I know your number!"

What is Claire's favorite number?

8. Equilateral triangle $\triangle ADG$, where the vertices are labeled clockwise, has side length 27. A circle is drawn such that it intersects each side of the equilateral triangle twice. Clockwise from A , label these intersections B, C, E, F, H , and I . If $AI = 6$, $GF = 7$, $DC = 15$, and the other six edges each have integer length, find $BC \cdot EF \cdot HI$.

9. Find the remainder when $\prod_{k=0}^{2026} (k^2 + 1)$ is divided by 2027.

10. For integers $n, m > 1$, the set of all divisors of n^2 that are NOT divisors of $m^2 - 1$ is the same as the set of divisors of $7n - 10$ that are NOT divisors of $m^2 + m$, and this set is nonempty. What is the sum of all possible values of n ?

11. Bella is decorating a new home for her pet bee, Betsy. The home is an 8×8 grid of cells. Bella has four distinct colors of paint to use. A set of four cells is called **colorful** if their centers form the corners of a rectangle with sides parallel to the grid edges, and all four cells are painted different colors. Bella wants to maximize the number of **colorful** sets to please Betsy. What is the maximum number of colorful rectangles she can achieve?

12. Triangle ABC has incircle ω and incenter I . Let Ω be the circle outside ABC that is tangent to side $\overline{BC}$ and to the extensions of the other two sides, and let J be its center. The circles ω and Ω touch $\overline{BC}$ at D and E , respectively. Suppose lines $\overleftrightarrow{IE}$ and $\overleftrightarrow{JD}$ meet at a point on ω . If ω and Ω have radii 12 and 25, respectively, then the perimeter of $\triangle ABC$ can be expressed as $\frac{m}{n}$, where m and n are relatively prime positive integers. What is $m+n$?

- 1.
- 2.
- 3.
- 4.
- 5.
- 6.
- 7.
- 8.
- 9.
- 10.
- 11.
- 12.

- 45 minutes
- No calculators
- Positive integer answers
- Write answers INSIDE the box

