Set 1

1. [**5**] Suppose real numbers x, y, z satisfy

$$x + y + 3z = 7$$
$$2x + 2y + 5z = 12$$

   What is the value of z?

2. [**5**] Tom has three straws of lengths 5, 6, 7, 8, and 9 inches, for a total of 15 straws. How many of his straws are shorter than at least half of all his other straws?

3. [**5**] Kathy and Grace are taking a 3 hour train ride. Kathy is asleep for exactly 2 hours of the train ride, and Grace is asleep for exactly 1 hour. If there are exactly 30 minutes of the train ride where both are awake, for how many minutes is exactly one of them awake?

. . . .

Set 2

4. [**6**] An is putting stickers on her laptop. She has four different stickers and wants to put at least one, but not all, on her laptop. How many ways can she do this? The order in which she places the stickers does not matter.

5. [**6**] Angelina has 100 flowers, each of which is red, pink, or purple. She creates 10 bouquets containing 10 flowers each. Each bouquet must have at least 5 purple flowers, and more pink than red flowers. What is the greatest number of red flowers she can have in total?

6. [**6**] Adeethyia wrote down a two-digit integer greater than 60, doubled it, subtracted 41, and got a two-digit positive integer with both digits identical. What is his number?

. . . .

Set 3

7. [**7**] A fraction in simplified form, with value less than 1, has its numerator and denominator sum to 36. The numerator and denominator are both positive integers. How many possible values are there for the fraction?

8. [**7**] Amy has 40 mittens, 14 of which are brown, 16 are red, and 10 are gold. She selects at random in the dark. What is the smallest number of mittens Amy must select to guarantee 2 matching pairs of different colors?

9. [**7**] The three longest diagonals in a regular hexagon of side length 1 are rearranged to form the sides of a triangle. Then, the six remaining diagonals are rearranged to form the sides of a new regular hexagon. What is the ratio of the area of the triangle to the area of the new hexagon?

Set 4

10. [**8**] Let m be the nearest integer multiple of 101 to 10,000,000,000 and n be the nearest integer multiple of 99 to 10,000,000,000. Find the difference $|m - n|$.

11. [**8**] Define the symbol
$$a \oplus b = (a \bmod b) + (b \bmod a),$$
where $a \bmod b$ is the remainder of a when divided by b. What is the value of $2 \oplus 3 \oplus \cdots \oplus 2026$, evaluated from left to right?

12. [**8**] Two regular tetrahedra (triangular pyramids) of equal size are placed adjacent edge-to-edge on a flat surface. If their edges have length 1, what is the distance between their top vertices?

Set 5

13. [**9**] A positive integer's odd divisors (including 1) add up to an odd positive integer n. What is the sum of the four smallest possible values of n?

14. [**9**] You have a row of 20 balls: two labeled 1, two labeled 2, up to two labeled 10, arranged in ascending order (1, 1, 2, 2, ..., 10, 10). Every turn, you can swap two adjacent balls. What is the minimum number of turns you need to take to rearrange the 20 balls in decreasing order?

15. [**9**] Cayley wants to travel to a point A which is 100 meters away from her current location. Unfortunately, Cayley is a crab, so she can only travel in semicircular steps of arc length 1. What is the least total integer distance Cayley can travel, in meters, to get to point A?

Set 6

16. [**10**] Jenny has 4 different colored crayons and a 16×16 unit grid. On the grid, she colors each edge of length 1 with a random color. Estimate the number of (not necessarily 1×1) squares, on average, with edges in the grid that have all four sides the same color. Express your answer as a base 10 decimal.

    If you are within 5% of the right answer, you get 10 points. Every additional 2% error deducts one point (minimum zero points).

17. [**10**] Richard and Yifan take turns picking balls out of a bag without replacement, with Richard drawing first. Richard wins if he picks a green ball before Yifan picks a purple ball. Suppose there are 2 green, 1 purple, and 999 yellow balls in the box. Estimate the probability that Richard wins. Express your answer as a base 10 decimal.

    If you are within 5% of the right answer, you get 10 points. Every additional 2% error deducts one point (minimum zero points).

18. [**10**] Determine whether each of the following statements are true or false.

    1. $6 \times 7 > 67$
    2. $67 \times 69 > 68^2$
    3. $6767 > 67 \times 67 + 6 \times 7 \times 6 \times 7$
    4. $6^7 > 7^6$
    5. $67^{76} > 76^{67}$
    6. $10^{225} > 225^{100}$
    7. $10^{250} > 250^{100}$
    8. $101^{102^{103}} > 103^{102^{101}}$
    9. $67! \cdot 68! \cdot 69! > 180!$
    10. $100^{250!} > 250^{100^{250}}$

    Submit a 10-tuple of letters corresponding to the answers above (where T denotes that the inequality is true, F denotes that it is false, and X denotes that you wish to leave the question blank).

    Your score is the number of correct answers minus the number of incorrect answers (minimum zero points).

. . . .

Set 7

19. [**11**] Let $\triangle ABC$ be an equilateral triangle with side length 1. Construct its circumcircle and select point D from minor arc AC such that $AD \times DC = \frac{1}{26}$. Compute $AD^2 + CD^2$.

20. [**11**] Jonathan bought an egg cooker that can cook eggs in batches of at most 6 eggs. He can buy eggs in packs of 10 or 14. Suppose that he bought n packs of eggs, cooked $2n - 3$ batches, and had 2 eggs left over, where n is a positive integer. What is the least possible value of n?

21. [**11**] Let $ABCD$ be a square, and furthermore, let ω_1 be the semicircle in $ABCD$ with diameter AB, and ω_2 be the quarter-circle in $ABCD$ centered at B with radius AB. A line passing through B intersects ω_1, ω_2, and $\overline{AD}$ at E, F, and G, respectively. Suppose $BE = 5$ and $FG = 2$. What is EF?

Set 8

22. [**12**] Michael randomly chooses a complex number z inside the unit disk in the complex plane. What is the probability that the real part of $(z-1)^3$ is positive?

23. [**12**] Leo has 10 distinct positive integers $n_1, n_2, \ldots, n_{10}$ such that each number divides the product of the other numbers. What is the smallest possible value of $n_1 + n_2 + \cdots + n_{10}$?

24. [**12**] Let $\triangle ABC$ be a triangle with $AB = 5, BC = 13$, and $\angle ABC = 2\angle ACB$. Compute the area of $\triangle ABC$.

. . . .

Set 9

25. [**13**] Let ABC be an isosceles triangle with $AB = AC$. Let M be the foot of the altitude from A to $\overline{BC}$, and let D be the foot of the altitude from C to $\overline{AB}$. Let H be the intersection of these two altitudes. Given that $AH = HM = 2$, compute MD.

26. [**13**] As all gacha game players know, the fifth perfect number P_5 is $2^{12}(2^{13}-1) = 33550336$, meaning that all proper divisors of P_5 (divisors not equal to itself) sum to P_5. Given $2^{13}-1$ is prime, how many positive integers n satisfy $n < P_5$, and three distinct proper divisors of n multiply to P_5?

27. [**13**] Let $(g_n)_n$ be a geometric sequence, and $(a_n)_n$ be a sequence of real numbers such that $a_1 = a_4$ and $|a_n - g_n| \le 4$ for all positive integers n. Let $r = a_3 - a_2$. What is the difference between the least and greatest possible values of r?

. . . .

Set 10

28. [**15**] Find the sum of all positive integers n satisfying

$$\left|\sum_{x=1}^{n} \log_{28}(x^2+7x+12)^{\cos(\pi x)}\right| = 1.$$

29. [**15**] Let $f(x) = x^3 + ax^2 + bx + c$, where a, b, c are distinct nonzero integers. If $f(a) = a^3$ and $f(b) = b^3$, find the value of c.

30. [**15**] Consider the sequence starting with $1, 2, 3$, and every subsequent term is the sum of three previous numbers in the sequence, randomly chosen without replacement. What is the expected value of the 200th term in this sequence?

Set 11

31. [**17**] $ABCD$ is a rectangle, E lies on AD in between A and D. Let l be the perpendicular bisector of line segment EC. Lines l and EC split $ABCD$ into four regions whose areas in some order are $52, 92, 117$, and 369. What is the perimeter of $ABCD$?

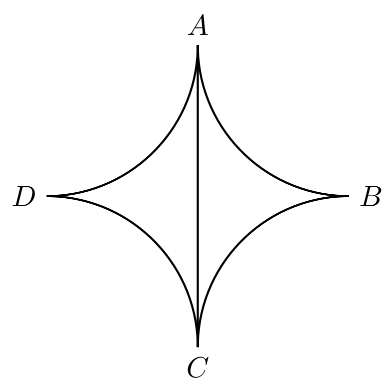
32. [**17**] Sebastian has an 8×8 checkerboard which consists of black and white squares. Every turn, they draw a square grid on the checkerboard, and swap the color of all squares inside that grid. What is the fewest number of turns they need to change every square to the same color?

33. [**17**] Suppose positive integers a, b, and c satisfy $4^a = b^4 + c! - 18$ and $a + b + c = S$. What is the sum of all possible values of S?

. . . .

Set 12

34. [**20**] The following shape $ABCD$ is created from the space between four tangent unit circles. 299 line segments with endpoints on the shape lie parallel to each other and to segment AC, and split $ABCD$ into 300 pieces of equal area. Estimate the sum of the lengths of all these segments. Express your answer as a base 10 decimal.



If you are within 2% of the right answer, you get 20 points. Every additional 1% error deducts one point (minimum zero points).

35. [**20**] Estimate:

$$\sum_{a=1}^{\infty}\sum_{b=1}^{\infty}\frac{ab - 2|a-2b| + \max(a,b)}{2^a + 2^b}.$$

Express your answer as a base 10 decimal.

If you are within 2% of the right answer, you get 20 points. Every additional 1% error deducts one point (minimum zero points).

36. [**20**] Let $L_6(n)$ be the number formed by replacing the last digit of n with 6. Let $F_7(n)$ be the number formed by replacing the first digit of n with 7. Suppose $\frac{L_6(n)+F_7(n)}{n}$ is an integer. Additionally, n does not contain the digits 6 or 7, but n is divisible by 6 or 7 (or both).

Find the set S of the 10 smallest positive integers n that satisfy these properties. If you list m distinct integers, and they are all in S, you get $2m$ points.