Individual Round

Rules
For the individual round, you will have 75 minutes to solve 15 short-answer questions. You are allowed to use writing utensils and scratch paper, but you may not use books, notes, calculators, slide rules, abaci, or any other computational aids. Similarly, you may not use graph paper, rulers, protractors, compasses, architectural tools, or any other drawing aids. You must complete this round individually, and we ask that you leave a seat between you and your neighbors.

Scoring
For each problem you solve correctly, you will be awarded 1 point. There is no penalty for guessing.

Answer Format
Acceptable answers include integers, fractions, decimals, exponents, and radicals. You may also use any basic arithmetic operations, such as addition, subtraction, multiplication, and division, to express your answers. Summation or product notation will not be accepted.

Answers must be simplified in order to receive credit. For example, fractions must be in lowest terms, and radicals should not have any square factors inside. Only exact answers will be accepted; in particular, decimal approximations will not receive credit.

Protests
If you have a protest, email brumo@brown.edu after the contest. All individual round protests must be submitted through email by 1 pm EST.

Reminders
Please fill in your full name, team name, and ID on the answer sheet. Write your answers clearly in the corresponding box on the answer sheet. If you need scratch paper, please raise your hand and we will bring it to you.

1. How many 3 digit numbers containing only the numbers $1, 2, 3$ and 4 as digits are greater than the number formed by reversing its digits?

2. 15 friends want to play a game 3 times. Each game seats exactly 10 players, and the other 5 must sit out. They want to make sure everyone plays at least once. What is the smallest possible number of people who must play 2 or more games for all 15 friends to play at least once across the three games?

3. Ethan is at the point $(7, 24)$ in the xy-plane. After being reflected across a line, Ethan is at the point $(8, 6)$. What is the y-intercept of the line of reflection?

4. This fall, Alice is busy raking the leaves in her yard. If no new leaves fall, she can finish the job in x hours. However, when leaves start falling steadily from the trees, it takes her $x + 2$ hours to finish. Later, her mischievous neighbor Malice begins secretly tossing in more leaves at the same rate that they fall from the trees, and now it takes Alice $x + 5$ hours to get the yard clean. What is the value of x?

5. In stormy cyber-Providence, William glances up at a holo-billboard that shows a four-digit positive integer N. Suddenly, the billboard is struck by lightning and glitches out: the integer gets divided by 9, then its digits get reversed, but the display still shows the same number N. What is N?

6. Noboru uses equally sized unit matchsticks to create regular unit hexagons on a table. If hexagons can share sides, what is the maximum number of unit hexagons he can create with 89 matchsticks?

7. Two integers a, b are considered relatively prime when their greatest common divisor is 1. Let an integer x be considered "good" if and only if $(x^2 + x)^2$ is relatively prime to 75. How many good integers are there between 1 and 150, inclusive?

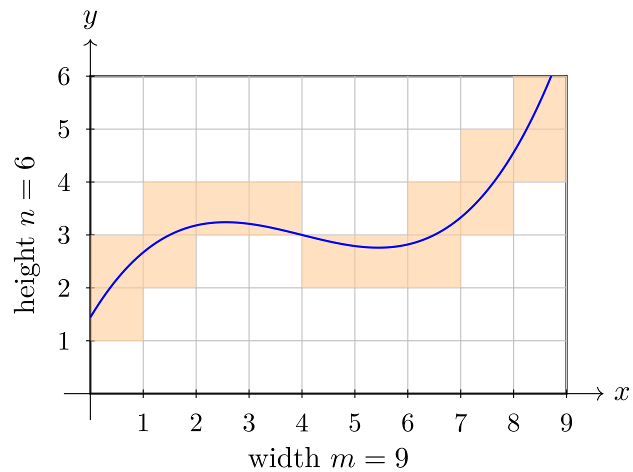
8. Let S be the set of integers n with $1 \leq n \leq 2025$ such that $\text{lcm}(n, 84) + \gcd(n, 84) = n + 84$. Compute the number of elements in S.

9. Let a, b, c be the three roots of the equation
$$x^3 + 6x^2 - 52x + 8 = 0.$$
Find the value of
$$\sqrt[3]{a} + \sqrt[3]{b} + \sqrt[3]{c}.$$

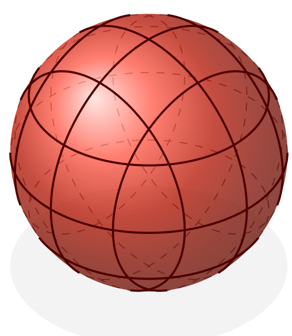
10. Let $ABCD$ be a convex quadrilateral such that $\angle BAD = 45^\circ$ and $BC = 4$. Let ω_1 be the circle centered at B passing through A and C and ω_2 be the circle centered at C passing through B and D such that ω_1 and ω_2 intersect at points P and Q, where P lies on segment AD and $P \neq D$. If lines AB and DC intersect at point X, the length of QX can be expressed as $\frac{a}{\sqrt{b}}$ for positive integers a and b, where b is not divisible by a square greater than 1. Find $a + b$.

11. Let $f(x)$ be a third degree monic polynomial over the integers; that is, $f(x) = x^3 + ax^2 + bx + c$ where a, b, c are integers. Suppose also that $0 \leq f(i) \leq 10$ for $i \in \{1, 3, 5\}$. Find the maximum possible value of $f(7)$.

12. In the xy-plane, a rectangular unit grid has width m and height n, parallel to the x and y axes respectively. We call a function "busy" if it intersects at least a third of the squares in the grid. What is the sum of all values of m such that for some n, there exists a "busy" cubic function but not a "busy" quadratic function? (Note that here, only intersecting a vertex of the square does not count as intersecting the square.)



13. Chef Joshua is dicing a perfectly spherical tomato on a rectangular cutting board. He makes 3 cuts parallel to the plane board, 3 cuts orthogonal to the length of the board, and the plane of the board. He makes the last 3 cuts orthogonal to the previous 6 for a total of 9 cuts. Given that every cut passes through some part of the tomato, the expected number of pieces Chef Joshua dices the tomato into is a number of the form $a + \frac{b\pi}{c}$. Calculate $a + b + c$.



14. For a given integer m, let $l_m(n)$ be the smallest integer $k \geq 1$ such that $n^{2^k} \equiv n \pmod{m}$, or 0 if no such k exists. For example $l_7(2) = 2$ as $2^{2^2} \equiv 2 \pmod{7}$. What is the maximum value of

$$\sum_{n=1}^{m} l_m(n)$$

among all two-digit positive integers m?

15. Triangle ABC has side length $BC = 3\sqrt{10}$. A circle ω with center O passes through the midpoint of $\overline{AB}$ and trisects $\overline{BC}$ at P and Q, where P is closer to B. X lies on ω such that B, A, X lie on the same line in that order. If $\angle AOQ = \angle BCX$ and $\angle BQO = \angle AXC$, what is the sum of the area of circle ω and the area of ABC?