BrUMO

Individual Round

Rules

For the individual round, you will have 75 minutes to solve 15 short-answer questions. You are allowed to use writing utensils and scratch paper, but you may not use books, notes, calculators, slide rules, abaci, or any other computational aids. Similarly, you may not use graph paper, rulers, protractors, compasses, architectural tools, or any other drawing aids. You must complete this round individually, and we ask that you leave a seat between you and your neighbors.

Scoring

For each problem you solve correctly, you will be awarded 1 point. There is no penalty for guessing.

Answer Format

Acceptable answers include integers, fractions, decimals, exponents, and radicals. You may also use any basic arithmetic operations, such as addition, subtraction, multiplication, and division, to express your answers. Summation or product notation will not be accepted.

Answers must be simplified in order to receive credit. For example, fractions must be in lowest terms, and radicals should not have any square factors inside. Only exact answers will be accepted; in particular, decimal approximations will not receive credit.

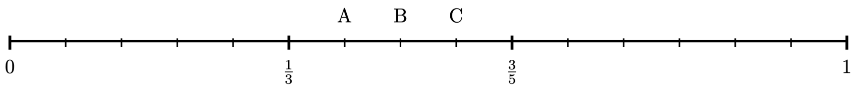
Protests

If you have a protest, email brumo@brown.edu after the contest. All individual round protests must be submitted through email by 1 pm EST.

Reminders

Please fill in your full name, team name, and ID on the answer sheet. Write your answers clearly in the corresponding box on the answer sheet. If you need scratch paper, please raise your hand and we will bring it to you.

1. Ashley (A), Bailing (B), and Claire (C) stand equally-spaced on a number line between the points $\frac{1}{3}$ and $\frac{3}{5}$, as shown below. Claire's position is given by the fraction $\frac{n}{15}$. What is the value of n?



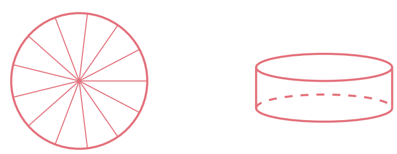
2. Bruno and Brunita are standing together in a line for a movie. They notice that the number of people ahead of them is seven times the number of people behind them. The total number of people in the line, including the two of them, is somewhere between 20 and 30. How many people are in the line?

3. Bruno has some number of nickels (5 cents), dimes (10 cents), and quarters (25 cents), possibly zero of any type. If he has 30 cents total, how many possible configurations of coins can he have (coin ordering does not matter)?

4. Lily and Coco each roll a fair 6-sided die. What is the probability that Lily's number is greater than Coco's number?

5. There are n red and m blue balls in a jar. If you increase the number of red balls by 20% of the total balls in the jar, then the number of red balls is 20% more than the number of blue balls. What is $\frac{n}{m}$?

6. Peter has a cake in the shape of a cylinder with height 2 in and base radius 5 in. He cuts the cake into 13 equally sized wedges. By how many in^2 does the surface area of the cake increase after Peter has made these cuts?



Top view   Side view

7. We have a list where every number n from 1 to 26 (inclusive) appears exactly n times (1 appears once, 2 appears twice, etc). What is the median of this list of numbers?

8. You are sending letters to friends A, B, and C. You have three letters, one written to each friend. Friends A, B, and C have a 50%, 40%, and 10% chance of responding if sent the right letter, respectively. All friends are guaranteed to respond if sent the wrong letter. You shuffle the letters and send one to each friend. What is the probability you get no response?

9. The BrUMO staff distributes scratch paper to 13 teams for the team round contest, giving the same amount of paper to every team while giving out as much paper as possible. When the BrUMO staff have n sheets of paper, each team receives m pieces, and there are 3 leftover. If the BrUMO staff instead distributed $10n$ sheets, each team would receive 32 pieces of paper (with some leftover). What is the value of $n + m$?

10. 15 friends want to play a game 3 times. Each game seats exactly 10 players, and the other 5 must sit out. They want to make sure everyone plays at least once. What is the smallest possible number of people who must play at least 2 games?

11. How many values of k is 20^{20} the least common multiple of $25^{10}, 10^{10}$, and k?

12. Every iDevice has a three-digit positive integer (without leading zeros) as its password. Unfortunately, Uruno's iDevice is glitchy, and every time he logs in, the middle digit of his password gets replaced with the average of the first and third digits. Uruno wants to choose a password that won't change even after the glitch. How many possible passwords are there for Uruno?

13. A circular room has radius 10. In the center of the room stands a circular pillar of radius 5. You and a lamp are each placed independently and randomly on the outer wall of the room (that is, on the circumference). What is the probability that you have a direct line of sight to the lamp (meaning you can draw a straight line from the lamp to you)?

14. In rectangle $ABCD$, let circle ω with center O outside $ABCD$ intersect the rectangle at points A and B. Suppose point P exists on side CD such that PA and PB are tangent to ω. If $\frac{OA}{AP} = \frac{1}{10}$, find $\frac{OA}{AD}$.

15. Ryan starts with an even two-digit positive integer (leading zeros are not allowed). He writes this number down and then repeatedly performs both steps in order:

    1. Divide the current number by 2. If the result is an integer, write it down. Otherwise, the process stops.
    2. Choose one digit of this number and replace it with the digit 8. Write down the result.

    At the end of the process, Ryan has written down k distinct numbers. What is the greatest possible value of k?