Redpoint Round

Rules

For the Redpoint round, you will have 45 minutes to solve 8 sets of 3 short-answer questions. You will choose 3 of these sets to submit, and these are the only sets that will be scored.

You are allowed to use writing utensils and scratch paper, but you may not use books, notes, calculators, slide rules, abaci, or any other computational aids. Similarly, you may not use graph paper, rulers, protractors, compasses, architectural tools, or any other drawing aids.

Scoring

For each problem you solve correctly in one of your selected sets, you will be awarded a number of points equal to the bolded number in the problem statement. There is no penalty for guessing. You may not submit the same set more than once.

Answer Format

Acceptable answers include integers, fractions, decimals, exponents, and radicals. You may also use any basic arithmetic operations, such as addition, subtraction, multiplication, and division, to express your answers. Summation or product notation will not be accepted.

Answers must be simplified in order to receive credit. For example, fractions must be in lowest terms, and radicals should not have any square factors inside. Only exact answers will be accepted; in particular, decimal approximations will not receive credit.

Protests

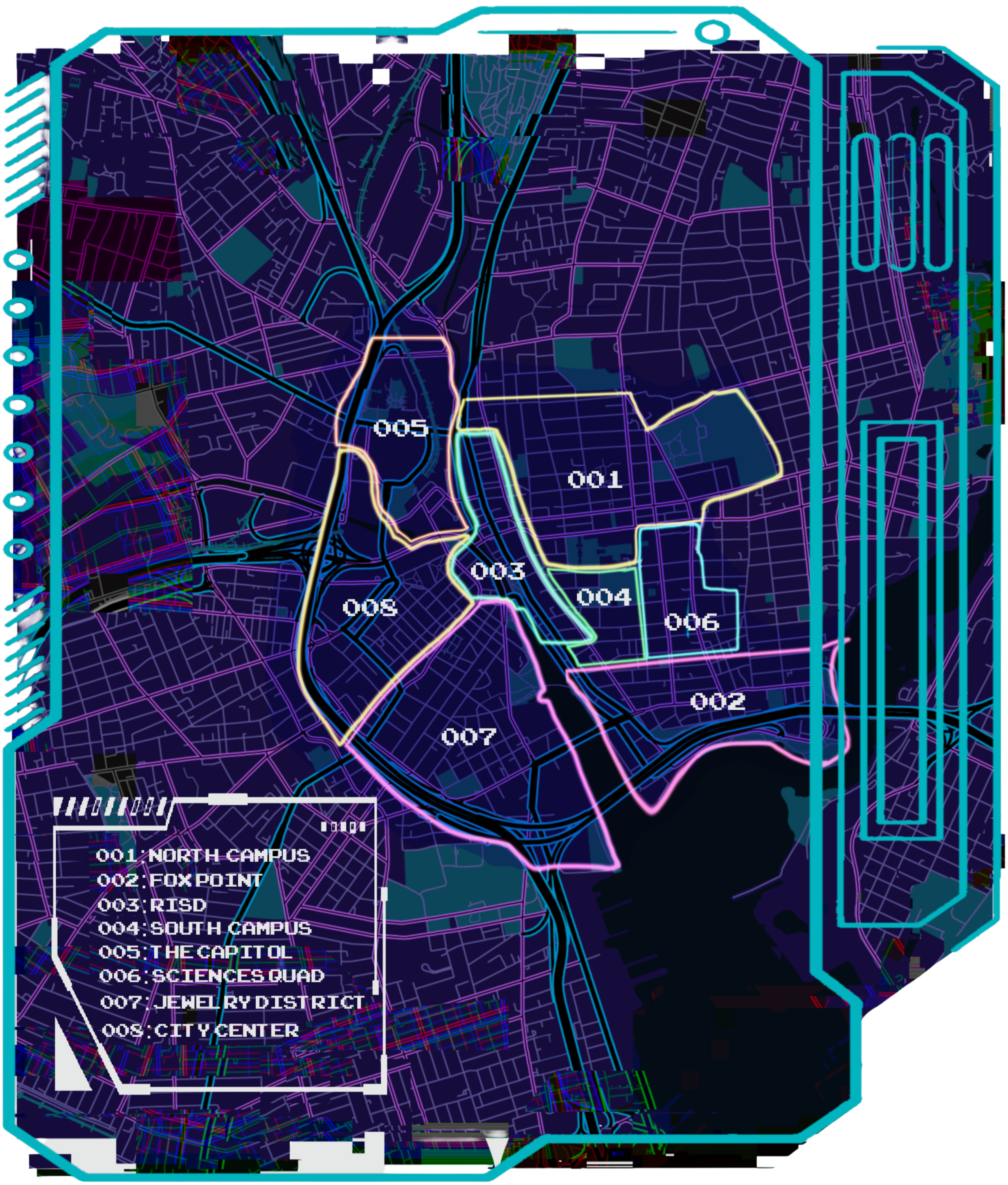
If you have a protest, email brumo@brown.edu after the contest. All Redpoint round protests must be submitted through email by 4 pm EST.

Reminders

Please fill in your team name and team ID on the answer sheet. Write your answers clearly in the corresponding box on the answer sheet. If you need scratch paper, please let your proctor know and we will bring it to you.

Dear BrUMO Competitors,

Disaster has struck! Cyber Providence has lost power due to a heavy snowstorm. Brown's campus and the surrounding area has been split up into 8 zones. Your mission is to choose 3 of these zones and solve the problems within them to help restore light to the city.



001: North Campus

1. [**6**] Vincent is rolling a modified 6-sided die. His die only has faces labeled with 1s and 2s. If the probability that he rolls a 1 is twice the probability that he rolls a 2, how many faces are labeled with 1?

2. [**6**] Faizaan's dorm number is a two-digit number. When you divide it by 7 or 9, the remainder is 2. What is his dorm number?

3. [**6**] Five first-year students take the MATH 350 midterm and receive positive integer scores. The mean test score is twice the median, and no one got the same score. What is the minimum possible value of the highest of their scores?

002: Fox Point

4. [**7**] Bruno needs his two-digit security code to withdraw a 1-dollar bill from an ATM at Eastside Marketplace. His code is the last two digits of $20^{20} + 26^{26}$. What two-digit number does Bruno need to enter?

5. [**7**] Bruno walks to Trader Joe's and pays for his groceries with a 1-dollar bill. He gets back 3 coins in change. He forgot the total price of his groceries, but he knows it's less than \$1 and contains at least one 7 in it. How many possible amounts could he have received in change?

6. [**7**] Back in his apartment, Bruno bakes a square pizza with area 9. He cuts it into 16 identical smaller squares. What is the sum of the perimeters of all of these squares?

003: RISD

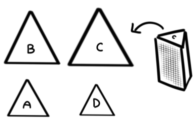
7. [**9**] Sunny draws a triangle with side lengths $8, 15$, and 17. He then draws a rectangle with the same perimeter and area as the triangle. If the diagonal of the rectangle has length L, what is L^2?

8. [**9**] Letters A_1, A_2, A_3, A_4, A_5, and A_6 sit in a circle in clockwise order. Letter A_1 starts with a paintbrush at minute 0. Every minute, the letter with the brush swaps their position with the letter sitting directly across from them, then passes the brush clockwise to the left. After 2026 minutes, for which i does A_i have the brush?

9. [**9**] Jake has 10 pastels, each either red or blue. He then adds an unknown number (possibly zero) of red pastels such that the original probability of randomly picking a red pastel is equal to the probability of randomly picking a blue pastel after adding in the red pastels. What is the sum of all possible amounts of red pastels that Jake added?

004: South Campus

10. [**11**] Josiah chooses a random integer b from 0 to 10, inclusive. Sharpe then chooses a random integer c from 0 to 20, inclusive. What is the probability that $x^2 - bx + c$ has exactly one positive root? Express your answer as a fraction reduced to lowest terms.



11. [**11**] When viewed from above, the four towers of Grad Center appear to be similar, non-congruent triangles. They each have integer side lengths, and their perimeters add up to 143. What is the greatest possible length of the smallest triangle's longest side?

12. [**11**] Four juniors, A, B, C, and D are playing a game. Independently, they each randomly pick a point on the circumference of a circle. D is the game master, and whoever's point is closest to D's point wins (ties count as a loss). However, A decides to cheat, secretly looking at B and Cs points before choosing their own point to maximize their chance of winning. How many times more likely is A to win if they cheat compared to if they do not cheat?

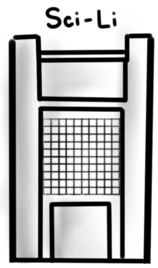
005: The Capitol

13. [**13**] Anthony wants to ride a bus that is supposed to come every 10 minutes, starting from 8:00AM. However, each bus has $\frac{1}{3}$ chance of not coming at all. If Anthony arrives at the bus station at a uniformly random time between 8:00AM to 9:00AM, what is Anthony's average wait time in minutes?

14. [**13**] There are 3 trains from Boston to Providence labeled T_1, T_2, and T_3, with exactly 2 students on each train. Each train independently has probability $\frac{1}{2}$ of being cancelled. If train T_i is cancelled, those students take the next uncancelled train T_j with $j > i$. If no such train exists, those students are stuck in Boston. The students are randomly divided into 2 teams of 3. Compute the probability that everyone on team 1 successfully reaches Providence.

15. [**13**] Outside the Capitol building, Efe polls a group of unknown size about ice cream flavors. The poll shows that 6% prefer strawberry, 7% prefer chocolate, 67% prefer vanilla, and the rest have no preference. However, Efe knows that the poll rounds percentages to the nearest integer (with 0.5 rounding up). What is the least possible number of people in the group that have no preference?

006: Sciences Quad



16. [**16**] The cloud service *Oscar* manages 20 virtual machines, each handling 100 requests. At time 0, there are no requests. Every second starting from time 1, 21 new requests arrive. Whenever total usage exceeds 50% capacity at time t, one new machine is queued up to be added at time $t+5$. From time t to $t+4$ (inclusive), no other computers are queued up. The system crashes at time T when it runs out of capacity. Compute T (in seconds).

    Note: if a machine is added at time t, it is added before the requests come at time t.

17. [**16**] Each machine in *Oscar* is in the shape of a convex quadrilateral $ABCD$ with $\angle A = 90°$ and $\angle B = 150°$. Furthermore, we are given that $\angle CAB = 18°$ and $AD = BC$. What is the measure of $\angle D$?

18. [**16**] In the SciLi, windows are arranged in a 10×10 grid. A screen is attached to each window, and a cable connects adjacent screens in each row and column. Screen A is at the top-left, and screen B is at the bottom-right. Bruno is playing a game and wants to move his character from A to B. He can move down or right to adjacent screens if they are connected by a cable.

    A recent meltdown destroyed one cable in such a way that maximized the number of remaining possible routes from A to B. Then, a party destroyed another cable in such a way that minimized the number of remaining possible routes from A to B. How many routes remain from A to B?

007: Jewelry District

19. [**20**] Let $f(x)$ be a function over the positive integers, such that for any two distinct positive integers m and n, the least common multiple of $f(m)$ and $f(n)$ is strictly less than $m+n$. Suppose $f(50) = 49$. How many possible values are there for the sum $f(1) + f(2) + f(3) + \cdots + f(49)$?

20. [**20**] Cody manufactures 16 rings in 8 pairs, with each pair having a different design. He arranges them such that 4 of the pairs are separated by an even number of rings and the other 4 are separated by an odd number. Let N be the number of possible arrangements of rings. If the prime factorization of N is $p_1^{n_1} p_2^{n_2} p_3^{n_3} p_4^{n_4}$, then calculate $p_1 + p_2 + p_3 + p_4 + n_1 + n_2 + n_3 + n_4$.

21. [**20**] A factory robot takes a jewel shaped as an acute triangle ABC and marks a point D such that
$$AB \cdot CD = AC \cdot BD = BC \cdot AD.$$
Given that $\angle ACB = 55°$, what is the value of $\angle ADB$ in degrees?

008: City Center

22. [**24**] For some reason, the route numbers for RIPTA buses at Kennedy Plaza are represented by complex numbers. Jaeho is trying to find the bus to Woonsocket, but he only knows that its route number is a complex number z such that the following equation has a real root:
$$x^2 + zx + 4 + 3i = 0.$$
Find the minimum possible value of $|z|$.

23. [**24**] A hockey puck of negligible radius sits on the perfectly circular Providence Rink, halfway between the center and the wall. Byron hits the hockey puck such that the 999th time it bounces off the circular wall is precisely at the point on the wall closest to where it started. How many possible directions could Byron have hit the hockey puck in?

24. [**24**] Let $a_0, a_1, a_2, \ldots, a_{150}$ be positive integers such that for all $n \geq 1$, the decimal expansion of $\frac{a_n}{a_{n-1}}$ contains the digits forming n directly after the decimal point. For example, $\left\lfloor \frac{a_{15}}{a_{14}} \right\rfloor = .15....$ What is the least possible value of a_{50}?