Team Round

Rules
For the team round, you will have 45 minutes to solve 10 short-answer questions. You are allowed to use writing utensils and scratch paper, but you may not use books, notes, calculators, slide rules, abaci, or any other computational aids. Similarly, you may not use graph paper, rulers, protractors, compasses, architectural tools, or any other drawing aids. You may collaborate with your teammates to solve these problems.

Scoring
For each problem you solve correctly, you will be awarded 1 point. There is no penalty for guessing.

Answer Format
Acceptable answers include integers, fractions, decimals, exponents, and radicals. You may also use any basic arithmetic operations, such as addition, subtraction, multiplication, and division, to express your answers. Summation or product notation will not be accepted.

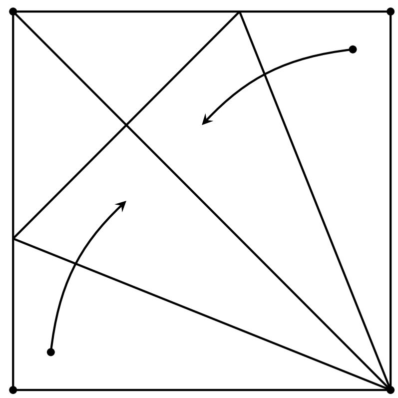
Answers must be simplified in order to receive credit. For example, fractions must be in lowest terms, and radicals should not have any square factors inside. Only exact answers will be accepted; in particular, decimal approximations will not receive credit.

Protests
If you have a protest, email brumo@brown.edu after the contest. All team round protests must be submitted through email by 1 pm EST.

Reminders
Please fill in your team name and team ID on the answer sheet. Write your answers clearly in the corresponding box on the answer sheet. If you need scratch paper, please let your proctor know and we will bring it to you.

1. Mason has $x^2 + 12x + 38$ apples for some positive integer x. When he splits these apples equally among $x + 2$ friends, he finds that there are 3 apples remaining. Find the sum of all possible values of x.

2. Jerry is folding paper cranes with square papers of side length 1. At one step, he has to form a crease from one corner to the opposite corner, and then fold two edges on either side of the diagonal crease (as shown in the diagram) so that they fully line up with the crease. What is the area of the resulting kite? Express your answer as $a\sqrt{b} + c$, where a, b, c are (not necessarily positive) integers and b contains no square factors other than 1.



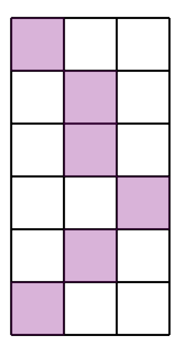
3. Amber is presented with the expression 1 ? 2 ? 3 ? 4 ? 5 ? 6 ? 7, where each question mark represents an operation. How many ways are there for Amber to fill in the six question marks with 2 '+'s , 2 '−'s, and 2 '×'s such that the result of the expression is an odd number?

4. A regular tetrahedron has a total surface area S_1. Let the surface area of its circumscribed sphere be S_2. It is known that
$$\frac{S_2}{S_1} = \frac{\sqrt{a}\pi}{b},$$
where a and b are positive integers with no common factors other than 1. Find $a + b$.

5. A megacorporation's vault performs a "format-flip" on a positive integer N: it writes N in base-3, then interprets that same string of digits as a base-4 numeral and converts it back to base-10. Denote the result by $F(N)$.

   If $F(N) - N = 2002$ in base-10, the vault unlocks. Among all possible code, the runner uses the smallest possible N to reduce detection time. Find N, with your answer written in base-10.

6. A rectangular grid has 6 rows and 3 columns. Two squares in the grid are said to be connected if they meet at a corner or an edge. If one square in each row is selected randomly, what is the probability that the 6 selected squares form a connected path?



7. Let $PUNK$ be a parallelogram with $PU = 24, PK = 70$, and $\angle UPK = 60^\circ$. Let Ω_1 be the circle passing through P and U tangent to $\overline{PK}$, and let Ω_2 be the circle passing through U and N tangent to $\overline{PU}$. Let Ω_1 and Ω_2 intersect at $C \neq U$. The length CP can be expressed as $\frac{m}{\sqrt{n}}$, where m and n are positive integers and n is not divisible by a square greater than 1. Compute $m + n$.

8. Calculate:

$$\sum_{n=0}^{\infty}\sum_{m=0}^{\infty} \frac{6(mn+1)^2 - (mn+n+m+2)^2}{2^n 3^m}$$

9. Let $f_1, f_2 \ldots f_{100}$ be real-valued functions such that for all positive integers k such that $k \leq 100$, $f_k(x)$ is either $\lfloor x \rfloor$ or $\lceil x \rceil$. Define $f(x) = f_{100}(x \cdot f_{99}(x \cdot f_{98}(\ldots x \cdot f_1(x) \ldots)))$. Suppose that, for a real number y, there are exactly 111 different possible values of $f(y)$. What is the greatest possible rational value of y? Express your answer as a fraction $\frac{a}{b}$, where a and b are positive integers with no common factors other than 1.

10. How many pairs of positive integers (n, k), both of which are not greater than 1000, are there such that

$$\sum_{i=0}^{16} k^{n+i} \equiv \sum_{i=0}^{16} (n+i)^k \pmod{17}?$$