



Team Round

Rules

For the team round, you will have 45 minutes to solve 10 short-answer questions. You are allowed to use writing utensils and scratch paper, but you may not use books, notes, calculators, slide rules, abaci, or any other computational aids. Similarly, you may not use graph paper, rulers, protractors, compasses, architectural tools, or any other drawing aids. You may collaborate with your teammates to solve these problems.

Scoring

For each problem your team solves correctly, your team will be awarded 1 point. There is no penalty for guessing.

Answer Format

Acceptable answers include integers, fractions, decimals, exponents, and radicals. You may also use any basic arithmetic operations, such as addition, subtraction, multiplication, and division, to express your answers. Summation or product notation will not be accepted.

Answers must be simplified in order to receive credit. For example, fractions must be in lowest terms, and radicals should not have any square factors inside. Only exact answers will be accepted; in particular, decimal approximations will not receive credit.

Protests

If you have a protest, email brumo@brown.edu after the contest. All team round protests must be submitted through email by 1 pm EST.

Reminders

Please fill in your team name and team ID on the answer sheet. Write your answers clearly in the corresponding box on the answer sheet. If you need scratch paper, please let your proctor know and we will bring it to you.

1. Bruno is teleported into a virtual world and forgets what day of the week it is. He meets a mercenary and a hacker. The mercenary lies on Mondays, Tuesdays, and Wednesdays, and on the other days, they tell the truth. The hacker lies on Thursdays, Fridays, and Saturdays, and on the other days, they tell the truth. "Yesterday, I was lying," the mercenary tells Bruno. "So was I," follows the hacker. What day of the week is it? 1 corresponds to Monday, 2 corresponds to Tuesday, and so on, with 7 corresponding to Sunday.
2. Three numbers are gossiping about a fourth number. One says, "it's prime!" Another says, "it's one more than a square number!" A third says, "it's divisible by four!"
Given that exactly one of these fellows is lying, and the number lies between 30 and 50, what is the value of the fourth number?
3. Each row, column, and bolded 2×2 subgrid of the following Sudoku contains the numbers 1, 2, 3, and 4 exactly once. However, exactly one of the given squares has been corrupted and contains an incorrect number. What is the number currently contained in the corrupted square?

1	3		4
4			2
2	4		3

4. Among 5 people, the tallest is 140 cm and the shortest is 80 cm. If the median height is equal to the average height, what is the smallest possible sum of their heights?
5. A cryptarithm is a mathematical equation where each letter represents a unique digit and leading zeros are not allowed. What is the sum of all possible values for the letter A in the following equation?

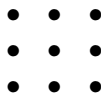
$$\begin{array}{rcccccc}
 & A & P & P & L & E \\
 + & A & P & P & L & E \\
 \hline
 O & R & A & N & G & E
 \end{array}$$

6. Bruno the cow grazes outside, tied to the corner of a rectangular shed measuring 3 feet by 8 feet with a 10-foot-long rope. What is the total area of ground, in square feet, that Bruno can graze on (not including the shed)? Leave π in your answer.
7. In the year 2052, our friend Mruno ends up working at the megacorporation Metapple. His job is to turn a special dial that only points to (possibly negative) integers. Then, he calculates the stability score using the formula $S(x) = x + \frac{12}{x+1}$, where x is the number his dial is pointing to. One day, he moves the dial from n to $n+1$. But oddly enough, he finds that the stability score doesn't change. What is the sum of all possible integer values of n ?

8. There are 9 lattice points (x, y) in the coordinate plane such that $|x| \leq 1$ and $|y| \leq 1$. How many quadratic equations

$$f(x) = ax^2 + bx + c$$

exist such that $f(x)$ goes through 3 of the points and $a > 0$?



9. The Salomon Auditorium contains a rectangular grid of chairs, with an unknown number of rows and columns. There are exactly 15 math students in each row and 9 computer science students in each column. If there are 7 empty seats, what is the smallest possible number of seats in the auditorium? Assume that no student studies both math and computer science.
10. An hourglass consists of two identical cones with tips vertically touching. Sand flows at a constant rate from the top cone to the bottom cone. The top cone starts with sand 12 cm deep, and a bit later, the level of sand in the top cone has decreased by 2 cm while that of the bottom cone has increased by 8 cm. Given that the top cone starts with 64 cm^3 of sand, how much sand is remaining in the top cone (in cm^3) when the bottom cone has filled up?

