2A *Time: 3 minutes*

Kristen has had her cat since it was a kitten. She said, "If you multiply my cat's age by 4, and then divide by 12, you get 5. How old is my cat?"

2B *Time: 4 minutes*

Amy had a nickel, a dime, a quarter, a half-dollar, and a silver dollar. After she lost one coin, she had exactly seven times as much money as her brother had. Which coin did she lose?

2C *Time: 5 minutes*

In a four-digit number, the sum of the thousands and hundreds digits is 3. The tens digit is 4 times the hundreds digit. The ones digit is seven more than the thousands digit. No two digits are equal. What is the four-digit number?

2D *Time: 5 minutes*

A square piece of paper is folded in half to form a rectangle. This rectangle has a perimeter of 24 cm. Find the area of the original square, in sq cm.

2E *Time: 6 minutes*

Boston is 295 miles from New York City along a certain route. A car starts from Boston at 1:00 PM and travels along this route toward New York at a steady rate of 50 mph. Another car starts from New York at 1:30 PM and travels along this route toward Boston at a steady rate of 40 mph. At what time do the cars pass each other?

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Answers: [2A] 15 [2B] Half-Dollar [2C] 2149 [2D] 64 [2E] 4:30

2A 66%

METHOD 1: *Strategy: Combine operations.*
If you multiply by 4 and then divide by 12, that is equivalent to dividing by 3. If her cat's age is divided by 3, the result is 5, so **her cat's age is 15.**

METHOD 2: *Strategy: Work backwards.*
A number divided by 12 is 5, so the number is 60. Then her cat's age multiplied by 4 is 60, so her cat's age is 15.

2B 49%

Strategy: Remove each coin, one at a time and check for divisiblity by 7.
The sum of all the coins is \$1.90. One coin can be removed in 5 different ways:

Coin removed	nickel	dime	quarter	half-dollar	dollar
Sum of remaining coins	1.85	1.80	1.65	1.40	0.90

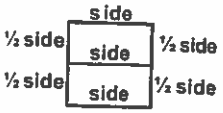
Only \$1.40 is divisible by 7. **Amy lost the half-dollar.**

2C 47%

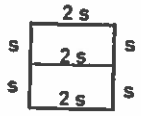
Strategy: Proceed one sentence at a time.
(1) The number is 1 2 _ _ or 2 1 _ _ or 3 0 _ _.
(2) The number is 1 2 8 _ or 2 1 4 _ or 3 0 0 _. Reject 3 0 0 _ (2 equal digits).
(3) The number is 1 2 8 8 or 2 1 4 9. Reject 1288 (2 equal digits).
The number is 2149.

2D 14%

METHOD 1: The length of the rectangle equals the length of one side of the square ("side"). The width of the rectangle ("½ side") is half the length of one side of the square.
Therefore the perimeter of the rectangle (given as 24 cm) is ½ + 1 + ½ + 1 = 3 times the length of one side of the square ("3 × side"). Then the length of one side of the square is 24÷3 = 8 cm, and **the area of the square is 64 sq cm.**



METHOD 2: *Strategy: Use algebra.*
To avoid the use of fractions, let 2s represent the length of one side of the square as shown. Then the perimeter of one of the congruent rectangles is 6s or 24. Thus s = 4, and the length of one side of the square is 2s or 8. **The area of the square is 64.**



METHOD 3: *Strategy: Make a table.*
Suppose the width of the rectangle is 1, 2, 3, and so on. Then find the length and perimeter. When the perimeter is 24, then find the area of the square.

width of rectangle	1	2	3	4
length of rectangle	2	4	6	8
perimeter	6	12	18	24

Thus, the length of one side of the square is 8 and the area of the square is 64.

Follow-up: Extend the question: what is the area if the paper is folded in thirds? quarters? fifths? sixths? To gain integral values for the length of one side, what relationship must exist between the number of rectangles and the given perimeter? To foster greater depth of understanding, what perimeter must be assigned if the paper is folded into 100 rectangles, and what will be the area of the square?

2E From 1:00 to 1:30, the car from Boston travels 25 miles. There are 270 miles left for the two cars to travel. From 1:30 until they pass each other, the car from Boston travels 50 miles each hour while the car from New York travels 40 miles each hour. Together they travel a total of 90 miles toward each other each hour. Thus, to travel 270 miles they need 3 hours starting at 1:30. **The cars pass each other at 4:30.**

Follow-up: Suppose each car traveled to Miami at the same rates and starting times, followed the same route, and spent the same amount of time on the road. (1) at what time would they meet? (2) How far would each travel? (3) consult a map: would they meet before they reach Miami? [4:30 PM – but a day later! They would meet 27 hours after the second car leaves. The cars would meet 1080 miles south of New York.]