OLYMPIAD 2 — DECEMBER 12, 2000

Answers: [2A] 357 [2B] 6754 [2C] 5 [2D] 300 [2E] 200

47%

2A Any number between 301 and 370 has a hundreds digit of 3. The sum of the digits is 5 x 3 or 15. Then the sum of the tens digit and the units digit is 15 - 3 or 12. The number is odd, so the units digit is odd. Test 339, 357, 375, and 393. 339 does not have 3 different digits. 375 and 393 are larger than 370. Only 357 satisfies all the conditions. **The number is 357.**

Follow-Ups on next page

45%

2B **METHOD 1:** *Strategy: Change subtraction into addition.*
The opposite of subtraction is addition. Add the difference and the subtrahend to find the minuend. Start with the ones: 8 + 6 = 14, so T = 4. Then 9 + 1 + Y ends in 5, so Y = 5. Since R = 7 and P = 6, **the number PR5T represents 6754.**
Check: 6754 – 4756 = 1998.

```
 1998
+47Y6
 PR5T
```

METHOD 2: *Strategy: Make a simpler problem.*
If we subtract 2 from 47Y6, the difference will be 2 more than 1998.
Then PR5T = 47Y4 + 2000 = 67Y4. Y must be 5, and PR5T = 6754.

```
 PR5T
-47Y4
 2000
```

41%

2C **METHOD 1:** *Strategy: Place the numbers around a circle.*
Since the first person in line goes to the end of the line, the line of people can be thought of as a circle of people. Write the numbers 1 through 10 around a circle. Beginning with the number 2, eliminate every second number. In order, eliminate 2, 4, 6, 8, 10, 3, 7, 1, 9. The last remaining number is 5. **The last remaining person occupied the 5th position originally.**

METHOD 2: *Strategy: Act out the movements.*

1 2 3 4 5 6 7 8 9 T	[10 people are in line. 1 is in front, and T (for 10) is in back.]
3 4 5 6 7 8 9 T 1	[1 goes to the back and 2 sits down.]
5 6 7 8 9 T 1 3	[3 goes to the back and 4 sits down.]
7 8 9 T 1 3 5	[And so on ...]
9 T 1 3 5 7	
1 3 5 7 9	
5 7 9 1	
9 1 5	
5 9	
5	The last remaining person was 5th in the original line.

15%

2D **METHOD 1:** *Strategy: Find the total surface area of the cube.*
A 10-cm cube has a surface area of 100 square centimeters on each of its 6 faces, making a total of 600 sq cm to be covered. To get an area of 600 sq cm of tape which is 2 cm wide, divide 600 sq cm by 2 cm. **The length of tape needed to cover the entire cube is 300 cm of tape.**

METHOD 2: *Strategy: Find the length of tape needed to cover one face of the cube.*
One face of a cube is 10 cm by 10 cm. To cover it with 2-cm tape, 5 pieces are needed, each 10 cm long. That is a total of 50 cm of tape. Since a cube has 6 faces, the length of tape needed to cover the entire cube is 6 × 50 = 300 cm of tape.

2E **METHOD 1:** *Strategy: First determine how much each new member paid.*
When 3 new members joined the club, the original five members each saved $15. This was a total savings of 5 x 15 or $75. The 3 new members paid a total of $75. Each new member paid 75 ÷ 3 or $25. All the members paid the same amount, so **the price of the used computer was** 8 x 25 = **$200.**

METHOD 2: *Strategy: Find the multiples of 5 and 8.*
Since both 5 people and 8 people can share the cost equally, the total cost has to be a multiple of both 5 and 8. The smallest multiple, 40, represents a saving of 15 ÷ 5 = $3 per person. To get a savings of 5 times that amount, multiply 40 x 5 for an original price of $200. Check: Five people sharing the cost equally is 200 ÷ 5 = $40. Eight people sharing the cost equally is 200 ÷ 8 = $25. The saving to each original member is 40 – 25 = $15.

METHOD 3: *Strategy: Find the multiples of 5 and 8.*
The computer costs 5 "old shares" or 8 "new shares".
Each new share is $15 less than an old share.
In dollars, 5 old shares = 8 old shares – 120 (that is, 8 x $15).
3 old shares = $120.
1 old share = $40.
The computer costs 5 old shares or $200.

FOLLOW-UP: Five members of a computer club buy a computer, dividing the cost equally. They each contribute $40. Later, one member leaves and wants her money back so the remaining members return her share. How much additional money does each member have to contribute? [$10]