OLYMPIAD 4

FEBRUARY 13, 2001

Answers: [4A] 1 [4B] 22 [4C] 11 [4D] A = 2, B = 1 [4E] 38

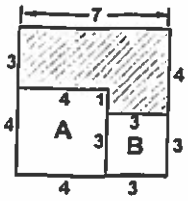
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4A Every 7 hours the hand returns to its starting point. After 7 x 3 or 21 hours, the hand points to 5. After 24 hours, the hand has moved 3 numbers past 5, and **24 hours from now, the hand points to 1.**

FOLLOW-UP: (1) If today is Friday, what day of the week will it be 24 days from now? 240? [Monday; Sunday] *(2) EXPLORATION: Experiment by replacing 5 and/or 24 with other numbers. What conclusions can students draw?*

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4B *Strategy: Add or subtract lengths of sides of the square to find the lengths of segments.*
Each side of the large square is 7 m.
The perimeter of the shaded region is 7 + 4 + 3 + 1 + 4 + 3.
The perimeter of the shaded region is 22 m.



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4C **METHOD 1:** *Strategy: Use the definition of an average.*
The sum of the original 6 numbers is 6 x 4 or 24. When the new number is added, the sum is 7 x 5 or 35. **The seventh number is** 35 – 24 = **11.**

METHOD 2: Suppose the seventh number is 4. Then the average of the seven numbers is still 4. To change the average to 5, add 1 to each of the seven numbers. Increasing the total by 7 is equivalent to adding 7 to the new number. Thus, the seventh number is 4 + 7 = 11.

FOLLOW-UP: Suppose 2 more numbers are added to the original 6. If the average of the 8 numbers is 10, what is the average of the two numbers that were added? [28]

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4D *Strategy: Fill in the squares one at a time.*
Step 1: From row 3, we see that C can't be 4 or 1. From column 2, C can't be 2. Therefore C must be 3.
Step 2: From row 3, D must be 2.
Step 3: From column 4, we see that E can't be 4 or 1. From the diagonal 1FDE, E can't be 1 or 2, since D is 2. Then E must be 3.
Step 4: From the diagonal, F must be 4.
Step 5: From column 2, B must be 1.
Step 6: From column 1, A can't be 1 or 4. From row 4, A can't be 1 or 3. So A must be 2. **Therefore, A = 2 and B = 1.**

1	2	3	4
	F		
4	C	D	1
A	B		E

FOLLOW-UP: In the problem, 6 squares were filled in to start you off. Can you fill in just four squares (one each of 1, 2, 3, and 4) so that all the other squares can be filled in uniquely according to the conditions of the problem? [Many possible answers exist. One is at the right.]

1			
		2	
			3
	4		

4E *Strategy: Work backwards.*

COINS IN BANK AFTER ACTION	ACTION	COINS IN BANK BEFORE ACTION
15 (Final result)	Took out 1/2 of the coins	30
30	Put in 4 coins	26
26	Took out 1/3 of the coins	39**
39	Put in 20 coins	19
19	Took out 1/2 of the coins	38 (Original amount)

To begin with, the bank contained 38 coins.

***Explanation of "39 coins":* After Sandy removed one third of the coins, the 26 remaining coins were two thirds of the amount before that removal. This number must be twice as large as the one third that she removed in this step. She must have removed 13 coins.