OUR SILVER ANNIVERSARY

OLYMPIAD PROBLEMS
2003 – 2004

DIVISION E

WITH
ANSWERS AND SOLUTIONS

25 Years Old ...

... From 1979 to 2004

Mathematical Olympiads for Elementary and Middle Schools

A Nonprofit Public Foundation

2154 Bellmore Avenue
Bellmore, NY 11710-5645

PHONE: (516) 781-2400
FAX: (516) 785-6640
E-MAIL: info@moems.org
WEBSITE: www.moems.org

NOTE: *Other problems related to some of those in this booklet can be found in our books "Math Olympiad Contest Problems for Elementary and Middle Schools" and "Creative Problem Solving in School Mathematics."*

Copyright © 2003 by Mathematical Olympiads for Elementary and Middle Schools, Inc. All rights reserved.

1A *Time: 3 minutes*

Kim stands in a line of people. She is the **25**th person counting from the front of the line. She is the 12th person counting from the rear. How many people are in the line?

1B *Time: 5 minutes*

There are 5 girls in a tennis class. How many different doubles teams of 2 girls each can be formed from the students in the class?

25 Years 1979 - 2004

1C *Time: 5 minutes*

At a fruit stand, an apple and a pear cost **25** cents, a pear and a banana cost 19 cents, and an apple and a banana cost 16 cents. Alex buys one apple, one pear, and one banana. How much does Alex spend?

1D *Time: 6 minutes*

A rectangle has a perimeter of 90 cm. The length of the rectangle is **25** cm more than its width. Find the number of sq cm in the area of the rectangle.

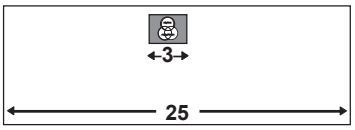
1E *Time: 6 minutes*

Find the sum of the DIGITS of the first **25** odd natural numbers.

Copyright © 2003 by Mathematical Olympiads for Elementary and Middle Schools, Inc. All rights reserved.

2A *Time: 3 minutes*

A picture 3 feet across hangs in the center of a wall that is **25** feet long. How many feet from the left end of the wall is the left edge of the picture?



25 Years 1979 - 2004

2B *Time: 4 minutes*

David buys a Beanie Baby. He later sells it to Jessica and loses $3 on the deal. Jessica makes a profit of $6 by selling it to Bryan for $**25**. How much did David pay for the Beanie Baby?

2C *Time: 5 minutes*

Ashley is twice as old as Carlos.
Billy is 5 years younger than Ashley.
The sum of the ages of the three children is **25**.
How old is Carlos?

2D *Time: 6 minutes*

The whole numbers from 100 down to 0 are arranged in columns P, Q, R, S, and T as indicated. Write the letter of the column that contains the number **25**.

P	Q	R	S	T
	100	99	98	97
93	94	95	96	
	92	91	90	89
85	86	87	88	
	84	83	82	81
77	78	79	80	
			and so on ...	

2E *Time: 6 minutes*

A cubical box without a top is 4 cm on each edge. It contains 64 identical 1-cm cubes that exactly fill the box. How many of these small cubes actually touch the box?

Copyright © 2003 by Mathematical Olympiads for Elementary and Middle Schools, Inc. All rights reserved.

3A *Time: 4 minutes*

T-shirts cost $8 each. Amy buys 1 T-shirt, Becky buys 3, Colin buys 5, Dan buys 7, and Emily buys 9. How many dollars do the five spend in total?

3B *Time: 5 minutes*

Find the sum of all natural numbers less than **25** which are ***not*** divisible by 2 or 5.

25 Years 1979 - 2004

3C *Time: 5 minutes*

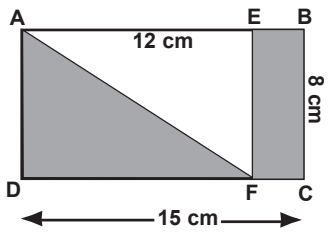
In a group of **25** girls, 8 run for the track team, 13 are on the math team, and 6 are on both teams. How many of the girls are not on either team?

3D *Time: 6 minutes*

I climb half the steps in a staircase. Next I climb one-third of the remaining steps. Then I climb one-eighth of the rest and stop to catch my breath. What is the smallest possible number of steps in the staircase?

3E *Time: 6 minutes*

ABCD and EBCF are both rectangles. The length of $\overline{CD}$ is 15 cm. The length of $\overline{BC}$ is 8 cm. The length of $\overline{AE}$ is 12 cm. Find the total number of sq cm in the areas of the shaded regions.



 Copyright © 2003 by Mathematical Olympiads for Elementary and Middle Schools, Inc. All rights reserved.

4A ***Time: 3 minutes***

Each of 6 piles contains a different number of pennies. Each pile contains at least one penny. What is the smallest possible total number of pennies in the 6 piles?

4B ***Time: 4 minutes***

The ages of Amanda, Brittany, and Carly are each prime numbers. Amanda is the youngest. The sum of the ages of Amanda and Brittany is equal to Carly's age. How old is Amanda?

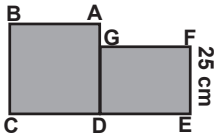
25 Years 1979 - 2004

4C ***Time: 5 minutes***

A bus can hold 24 adults or 30 children. **25** children are already on the bus. What is the largest number of adults that can still get on the bus?

4D ***Time: 6 minutes***

ABCD is a square. DEFG is a rectangle. The length of $\overline{EF}$ is **25** cm. The length of $\overline{DE}$ is the same as the length of $\overline{CD}$. The perimeter of the entire shaded region is 180 cm. Find the number of cm in the length of $\overline{AG}$.



4E ***Time: 7 minutes***

The average of a group of 20 numbers is 20. The average of a different group of 60 numbers is 60. The two groups of numbers are combined into a single group. What is the average of the combined group?

Copyright © 2003 by Mathematical Olympiads for Elementary and Middle Schools, Inc. All rights reserved.

5A ***Time: 4 minutes***

May **25**, 2025, will occur on a Sunday. On which day of the week will May 1, 2025, occur?

5B ***Time: 5 minutes***

The four-digit number *A*7*A*8 is divisible by 9. What digit does A represent?

25 Years 1979 - 2004

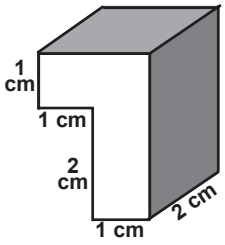
5C ***Time: 7 minutes***

In each turn of a certain game, only the following point-scores are possible: 5, 3, 2, 0. Eight turns are taken. In how many ways can the total point-score be **25**?

Do not consider the order in which points are scored.

5D ***Time: 7 minutes***

What is the largest number of these wooden "Els" that can be packed in a box that is 2 cm by 4 cm by 6 cm?



5E ***Time: 5 minutes***

Kayla has N marbles. She groups them by threes and has one left over. She groups them by sevens and has four left over. Kayla has more than five marbles. What is the smallest number of marbles Kayla could have?

 Copyright © 2003 by Mathematical Olympiads for Elementary and Middle Schools, Inc. All rights reserved.

ANSWERS AND SOLUTIONS

Note: Number in parentheses indicates percent of all competitors with a correct answer.

OLYMPIAD 1 — NOVEMBER 18, 2003

Answers: [1A] **36** [1B] **10** [1C] **30** [1D] **350** [1E] **175**

59% correct

1A *Strategy: Count each part of the line separately.*
There are 24 people in front of Kim and 11 people behind her. Counting Kim herself, **there are** 24 + 11 + 1 or **36 people in the line.**

48%

1B *Strategy: Make an organized list.*
Denote the girls as A, B, C, D, and E.

METHOD 1:

Teams including A:	AB, AC, AD, AE.
Teams including B but not A:	BC, BD, BE
Teams including C but not A or B:	CD, CE
Teams not including A, B, or C:	DE.

There are 4 + 3 + 2 + 1 or **10 different teams** that **can be formed.**

METHOD 2: *Strategy: Use Count the number of teams that include girls.*
As above, 4 teams include girl A. Likewise, 4 teams include B, 4 teams include C, 4 teams include D, and 4 teams include E. This list of 5 × 4 or 20 teams lists each team twice. For example, BC is listed as a team that contains B as well as a team that contains C. Therefore there are 20 ÷ 2 or 10 teams that can be formed.

40%

1C **METHOD 1:** *Strategy: Use the symmetry of the given information.*
Notice that each of the costs mentioned is for a different pair of fruits, and each type of fruit is mentioned twice. Suppose Alex buys an apple and a pear, then a pear and a banana, and then an apple and a banana. His purchase of ***two*** of each kind of fruit would cost 25 + 19 + 16 or 60¢. Then ***one*** of each would cost half as much, so **Alex spends 30¢.**

METHOD 2: *Strategy: Set up a chart. Guess and check.*

Suppose 1 apple costs:	8¢	9¢	10¢	**11¢**	12¢
1 apple and 1 pear cost 25¢, so 1 pear costs:	17¢	16¢	15¢	**14¢**	13¢
1 pear and 1 banana cost 19¢, so 1 banana costs:	2¢	3¢	4¢	**5¢**	6¢
From above, 1 apple and 1 banana cost:	10¢	12¢	14¢	**16¢**	18¢
Does 1 apple and 1 banana cost 16¢?	No	No	No	**Yes**	No

Alex spends 11 + 14 + 5 or 30¢.

METHOD 3: *Strategy: Determine the cost of one piece of each fruit.*
An apple and a pear cost 25¢, while a pear and a banana cost 19¢. Then one apple costs 6¢ more than one banana. So for 6¢ more than the cost of one apple and one banana, Alex could buy *two* apples. Then 2 apples would cost 16 + 6 or 22¢, so one

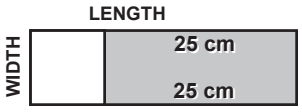
Copyright © 2003 by Mathematical Olympiads for Elementary and Middle Schools, Inc. All rights reserved.

apple costs 11¢. Since one apple and one pear cost 25¢, one pear costs 25 – 11 or 14¢, and one banana costs 19 – 14 or 5¢. Therefore Alex spends 14 + 11 + 5 or 30 cents.

Follow-Up: *A candy bar costs 25¢. If Amy, Beth, and Cara put their money together, they have 74¢. If Amy, Beth, and Dee put their money together, they have 72¢. Amy, Cara, and Dee have 71¢. Beth, Cara, and Dee have 68¢. If all 4 put their money together, how many candy bars can they buy?* [3- the total is 95¢]

31%

1D **METHOD 1:** *Strategy: Change the figure to a simpler shape.*
Reduce the length by 25 cm, leaving a square. The perimeter of the square is 2 × 25 or 50 cm less than the perimeter of the original rectangle. A square with a perimeter of 90 – 50 or 40 cm has a side-length of 10 cm. The width of the original rectangle is 10 cm and the length is 10 + 25 or 35 cm. **The area of the rectangle is** 10 × 35 or **350 sq cm.**



Alternatively, the width could be increased by 25 cm to form a square.

METHOD 2: *Strategy: Find the semiperimeter.*
The perimeter of a rectangle, the sum of two lengths and two widths, is 90 cm. The sum of one length and one width, called the *semiperimeter*, is half as large, or 45 cm. The two numbers with a sum of 45 that differ by 25 are 10 and 35. The area is 10 × 35 or 350 sq cm.

8%

1E *Strategy: Group conveniently and add the ones and tens digits separately.*
The first 25 odd natural numbers are 1, 3, 5, 7, 9, ... , 49. Arrange as shown in the table. Add the ones and tens digits separately in each column and then combine the subtotals.

The first 25 natural numbers	1	11	21	31	41	**TOTALS**
	3	13	23	33	43	
	5	15	25	35	45	
	7	17	27	37	47	
	9	19	29	39	49	
Sum of the ones digits	25	25	25	25	25	**125**
Sum of the tens digits	0	5	10	15	20	**50**
			Sum of all the digits			**175**

The sum of all the digits is 175.

Follow-Ups: ***(1)*** *What is the 10th odd natural number?* [19] *The 50th?* [99] *The 2003rd?* [4005] ***(2)*** *What is the sum of the first 5 odd natural numbers?* [25] *The first 6?* [36] *The first 100?* [10000]

Copyright © 2003 by Mathematical Olympiads for Elementary and Middle Schools, Inc. All rights reserved.

OLYMPIAD 2 — DECEMBER 16, 2003

Answers: [2A] **11** [2B] **22** [2C] **6** [2D] **T** [2E] **52**

58% correct

2A *Strategy: Determine how much "blank wall" shows.*

25 – 3 = 22. 22 feet of the width of the wall is blank. The picture hangs in the center, so each side of the picture has the same the amount of blank wall. 22 ÷ 2 = 11. **It is 11 ft from the left edge of the wall to the left edge of the picture.**

45%

2B *Strategy: Work backwards, using inverse (opposite) operations.*

Question: ? → (Subtract 3) → ☐ → (Add 6) → **$25**

Solution: $22 ← (Add 3) ← $19 ← (Subtract 6) ← $25

David paid $22 for the Beanie Baby.

74%

2C **METHOD 1:** *Strategy: Guess Carlos' age. With a table find the other ages.*

Choose values for Carlos' age. Use the facts that Ashley is twice as old as Carlos and Billy is 5 years younger than Ashley. For which value of Carlos' age is the sum of their ages 25?

	Carlos' age	Ashley's age	Billy's age	Sum	Sum = 25?
Trial 1	**4**	**8**	**3**	**15**	**No**
Trial 2	**5**	**10**	**5**	**20**	**No**
Trial 3	**6**	**12**	**7**	**25**	**YES**

Carlos is 6 years old.

METHOD 2: *Strategy: Use algebra.*

Let Carlos's age = C. Then Ashley's age = 2C and Billy's age = 2C - 5.

$C + 2C + (2C - 5) = 25$ Combine like terms.

$5C - 5 = 25$ Add 5 to each side.

$5C = 30$ Divide by 5.

$C = 6$ Carlos is 6 years old.

45%

2D **METHOD 1:** *Strategy: Look for a pattern.*

Each two successive rows contain eight numbers. Each number is therefore 8 less than the number two rows above it.

25 is 1 more than a multiple of 8. Every number that is 1 more than a multiple of 8 is in column T. Therefore **the number 25 belongs in column T.**

P	Q	R	S	T
	100	99	98	97
93	94	95	96	
	92	91	90	89
85	86	87	88	
	84	83	82	81
77	78	79	80	
			and so on ...	

Copyright © 2003 by Mathematical Olympiads for Elementary and Middle Schools, Inc. All rights reserved.

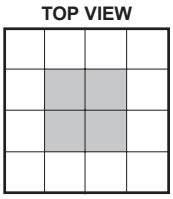
METHOD 2: *Strategy: Extend the table.*

P	Q	R	S	T
	100	99	98	97
93	94	95	96	
	92	91	90	89
85	86	87	88	
	84	83	82	81
77	78	79	80	
⋮	⋮	⋮	⋮	⋮
	36	35	34	33
29	30	31	32	
	28	27	26	25

Follow-up: *Suppose Day 1 is a Tuesday. What day of the week is Day 2003? [Tuesday]*

10%

2E **METHOD 1:** *Strategy: Count the cubes that do **not** touch a side or the bottom of the box.*
Consider each of the 4 horizontal layers in turn. In the bottom layer of 16 cubes, each small cube touches the bottom of the box. In each of the other 3 layers, the 4 center cubes do not touch a side of the box. There are 4 + 4 + 4 or 12 cubes that do not touch a side of the bottom of the box and therefore 64 – 12 or **52 of the small cubes touch a side or the bottom of the box.**



METHOD 2: *Strategy: Count the cubes in each horizontal layer that touch the box.*
In the bottom layer, all 16 cubes touch. In each of the other layers, the cubes along the edges touch a side of the box. There are 12 edge cubes in each of the top 3 layers that touch a side of the box, so 16 + 12 + 12 + 12 or 52 small cubes touch a side or the bottom of the box.

Follow-Ups:
(1) Suppose the box has a top. How many of the cubes do not touch the box? [8]
(2) A covered box 6 cm on a side is completely filled with 1-cm cubes. How many do not touch the box? [64]
(3) A covered box 12 cm on a side is completely filled with 1-cm cubes. How many do not touch the box? [1000]
(4) A covered box is filled with 1 cm cubes. Only one of them does not touch the box. How many do touch the box? [26]

OLYMPIAD 3 JANUARY 13, 2004

Answers: [3A] **200** [3B] **124** [3C] **10** [3D] **24** [3E] **72**

Copyright © 2003 by Mathematical Olympiads for Elementary and Middle Schools, Inc. All rights reserved.

76%

3A **METHOD 1:** *Strategy: Find how much each person spends.*
At $8 per shirt, Amy spent $8 for one shirt, Becky spent $24 for 3 shirts, Colin spent $40 for 5 shirts, Dan spent $56 for 7 shirts and Emily spent $72 for 9 shirts. **The five people spend $200 in total.**

METHOD 2: *Strategy: Find the total number of shirts.*
1+3+5+7+9 = 25 shirts are bought.
25 shirts @ $8 each costs $200
The five spent $200 in total.

(Note that the two methods are linked by the distributive property.)

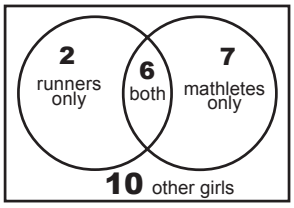
FOLLOW-UPS: *An outfit consists of a shirt and a pair of pants. Shirts cost $8 each, and pants $24 each. What is the largest number of outfits Amy can buy for $100 if* ***(1)*** *each shirt can be worn with just one pair of pants?* [3] ***(2)*** *each shirt can be worn with any pair of pants?* [12]

46%

3B *Strategy: Make a list.*
List all the natural numbers less than 25, from 1 to 24.
Remove those divisible by 2 or 5. Then 1, 3, 7, 9, 11, 13, 17, 19, 21, 23 remain.
Their sum is 124.
The sum of all natural numbers less than 25 which are not divisible by 2 or 5 is 124.

24%

3C **METHOD 1:** *Strategy: Use a Venn Diagram.*
Of the 8 runners, 6 are also mathletes. Then 2 girls are runners only. Of the 13 mathletes, 6 are also runners. Then 7 girls are mathletes only. Thus, a total of 6+2+7 = 15 girls participate in at least one of the two activities. Of the total number of 25 girls, **10 girls are not on either team.**



METHOD 2: *Strategy: Combine the teams but remove those counted twice.*
There are 13 mathletes and 8 runners. Of the 13 + 8 or 21 girls, 6 were counted twice. There is a total of 21 – 6 or 15 girls on the teams, so 25 – 15 or 10 girls are not on either team.

FOLLOW-UP: *Suppose 3 of the girls who are on both the math and basketball teams are also delegates to the student council, and only 5 of the girls in the original group of 25 are not on either team or on the student council. How many girls are only on the student council?* [5]

32%

3D **METHOD 1:** *Strategy: Work backward, considering the numbers of steps.*

Third climb: The smallest number of possible steps is 1. If the one step I climb is $\frac{1}{8}$ of the remaining steps, then 8 steps remain after the second climb.

Second climb: I climb $\frac{1}{3}$ of the remaining steps, so $\frac{2}{3}$ of the steps are left. Since $\frac{2}{3}$ of those steps are 8 steps, then $\frac{1}{3}$ of those remaining steps are 4 steps. Then 12 steps remain after the first climb.

First climb: I climb half the total number of steps and 12 steps remain. Therefore, **the smallest possible number of steps in the staircase is 24.**

Copyright © 2003 by Mathematical Olympiads for Elementary and Middle Schools, Inc. All rights reserved.

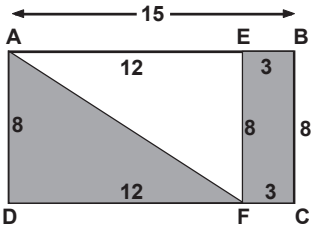
METHOD 2: *Strategy: Work forward, considering the fractions.*
I first climb $\frac{1}{2}$ the steps, so $\frac{1}{2}$ the steps remain.

I then climb $\frac{1}{3}$ of the remaining steps. $\frac{1}{3}$ of $\frac{1}{2}$ is $\frac{1}{6}$ of the total number of steps. I have now climbed $\frac{1}{2} + \frac{1}{6}$ or $\frac{2}{3}$ of the steps. Therefore $\frac{1}{3}$ of the steps remain.

Finally, I climb $\frac{1}{8}$ of these steps. $\frac{1}{8}$ of $\frac{1}{3}$ is $\frac{1}{24}$ of the total number of steps. The smallest number of steps possible is 1. If 1 is $\frac{1}{24}$ of the total number of steps, the smallest number of steps in the staircase is 24.

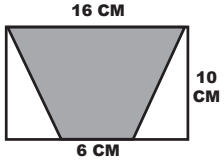
22%

3E **METHOD 1:** *Strategy: Find the area of the region that is not shaded.*
The area of rectangle ABCD = 15 × 8 = 120 sq cm.
The area of rectangle AEFD = 12 × 8 = 96 sq cm.
The area of unshaded triangle AEF = half of 96 = 48 sq cm.
The area of the shaded region = area of ABCD – area of AEF.
The total area of the shaded regions is 120 – 48 = **72 sq. cm.**



METHOD 2: *Strategy: Add the areas of the shaded regions.*
FC = 15 – 12 or 3 cm.
Area of rectangle EBCF = 3 × 8 or 24 sq. cm.
Area of triangle ADF = $\frac{1}{2}$ of the area of rectangle AEFD.
Then $\frac{1}{2}$ of 12 × 8 = 48.
The area of the shaded region is the sum of the areas of rectangle FCBE and triangle AFD.
The total area of the shaded regions is 24 + 48 = 72 sq. cm.

FOLLOW-UP: *Find the area of the shaded region.* [110 sq cm]



OLYMPIAD 4 FEBRUARY 10, 2004

Answers: [4A] **21** [4B] **2** [4C] **4** [4D] **5** [4E] **50**

70% correct

4A *Strategy: Draw a picture.*
The smallest number of pennies in one pile is 1. The next smallest possibility is 2, and so on.



The sum of 1 + 2 + 3 + 4 + 5 + 6 = 21. **The smallest possible total is 21.**

FOLLOW-UPS on next page.

 Copyright © 2003 by Mathematical Olympiads for Elementary and Middle Schools, Inc. All rights reserved.

FOLLOW-UPS: *There are 32 pennies in 6 piles. Each pile has a different number of pennies. What is the largest possible number of pennies in the largest pile?* [17] *What is the largest possible number of pennies in the smallest pile?* [2]

53%

4B *Strategy: Use knowledge of prime numbers.*
At least one of the ages must be even, because an odd number added to an odd number gives an even sum. The only even prime number is 2. Since 2 is also the smallest prime number, it must be Amanda's age. **Amanda is 2 years old.**

FOLLOW-UP: *Suppose Carly is less than 50 years old. What is the greatest possible age she could be?* [43]

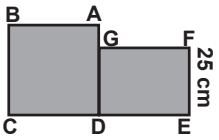
29%

4C **METHOD 1:** *Strategy: Use a ratio.*
24 adults require the same space as 30 children. Simpifying, 4 adults require the same space as 5 children. Since 5 more children could get on the bus to make 30 children, and 5 children require the same space as 4 adults, **4 adults can still get on the bus.**

METHOD 2: *Strategy: Use fractions.*
The bus holds 30 children and 25 children are on the bus. Then $\frac{25}{30}$ of the space on the bus is used. $\frac{25}{30}$ reduces to $\frac{5}{6}$. Since $\frac{5}{6}$ of the space is occupied, then $\frac{1}{6}$ of the total space remains for adults. The bus holds 24 adults, so $\frac{1}{6}$ of them, or 4 adults, can still get on the bus.

41%

4D *Strategy: Find the length of a side of the square.*
AB + BC + CD + DE + EF + FG + GA = 180.



AB = BC = CD because they are sides of square ABCD.
DE = GF because they are opposite sides of rectangle DEFG.
DE = CD, so AB = BC = CD = DE = GF.

FE = GD, so AG + FE is the same as AG + GD, another side of square ABCD.

Then the sum of 6 equal lengths is 180 cm. Each of them must be 30 cm. AG + 25 = 30, so **the length of AG is 5 cm.**

14%

4E **METHOD 1:** *Strategy: Find the total of each group of numbers.*
The average of a group of items is their total divided by the number of items in the group. Since the average of 20 numbers is 20, the sum of the 20 numbers is 400 (20 × 20). The average of the 60 numbers is 60, so their sum is 3,600 (60 × 60). When the two groups are combined, the new group has 80 numbers and the sum of all the numbers is 400 + 3600 or 4000. **The average of the combined group is** 4000 ÷ 80 or **50.**

(Note: Because the groups are of different size, the average of the combined groups is ***not*** *the same as the average of the averages.)*

METHOD 2: *Strategy: Use the ratio of the sizes of the two sets of numbers.*
The combined group has 80 numbers. $\frac{1}{4}$ of them (20 out of 80) are in the first group and $\frac{3}{4}$ of them (60 out of 80) are in the second group. Then the average of the combined group (called a *weighted average*) is nearer to the average of 60 than it is to the average of 20, and $\frac{1}{4}$ of the interval from 60 to 20. $\frac{1}{4}$ of 60 – 20 = $\frac{1}{4}$ of 40 = 10. Thus the weighted average is 60 – 10 or 50.

Note that method 2 may be too sophisticated for some elementary students.

FOLLOW-UP on next page.

Copyright © 2003 by Mathematical Olympiads for Elementary and Middle Schools, Inc. All rights reserved.

FOLLOW-UP: *In the third marking period, the average of Kelsie's 7 math tests is 89. What grade does she need on the 8th test to raise her average to exactly 90?* [97]

OLYMPIAD 5 — MARCH 9, 2004

Answers: [5A] **Thursday** [5B] **6** [5C] **7** [5D] **6** [5E] **25**

70% correct

5A **METHOD 1:** *Strategy: Count backward from the first Sunday.*
One week equals seven days. Earlier Sundays are May 18, 11, and 4. Therefore May 3 occurs on Saturday, May 2 occurs on Friday, and **May 1 occurs on Thursday.**

METHOD 2: *Strategy: Count forward from May 1.*
May 1 is on the same day of the week as May 8, 15, and 22. May 22 is 3 days before May 25, and 3 days before Sunday is Thursday. May 1 occurs on a Thursday.

FOLLOW-UPS: *(1) Suppose there were only 5 days in a week* (SMTWTh) *and the 25th was on a Sunday. On what day would the 1st be?* [M] *(2) How many days could there be in a week for the 1st and the 25th to occur on the same day of the week?* [any factor of 24]

62%

5B *Strategy: Use the test for divisibility by 9.*
If a number is divisible by 9, the sum of its digits is divisible by 9.
Then A + 7 + A + 8 must be 9 or 18 or 27 or 36.
A + A + 15 cannot be 9, which is too small or 36, which is too large.
If A + A + 15 = 18, A would not be a digit.
Since A + A + 15 = 27, **A represents 6.**

11%

5C *Strategy: Start with fives and replace with twos and threes.*

Start with:	**five 5s, three 0s**	5	5	5	5	5	0	0	0

Remove:	**Replace with:**	**Result:**							
one 5, one 0	one 2, one 3	5	5	5	5	**2**	**3**	0	0
two 5s, two 0s	two 2s, two 3s	5	5	5	**2**	**2**	**3**	**3**	0
two 5s, three 0s	five 2s	5	5	5	**2**	**2**	**2**	**2**	**2**
three 5s, two 0s	five 3s	5	5	**3**	**3**	**3**	**3**	**3**	0
three 5s, three 0s	three 2s, three 3s	5	5	**2**	**2**	**2**	**3**	**3**	**3**
four 5s, three 0s	one 2, six 3s	5	**2**	**3**	**3**	**3**	**3**	**3**	**3**

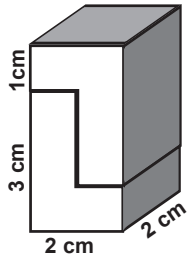
To replace five 5s requires more than 8 scores since 8 × 3 is only 24.

There are 7 combinations that give 25 points in 8 plays.

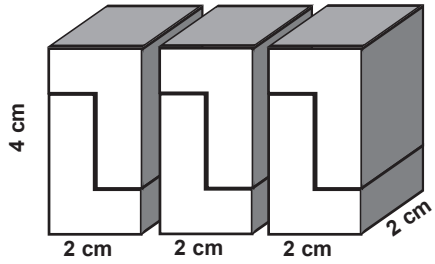
Copyright © 2003 by Mathematical Olympiads for Elementary and Middle Schools, Inc. All rights reserved.

19%

5D *Strategy: Combine pieces to form more familiar shapes.*
Two of the "Els" can be placed next to each other to form a 2 x 4 x 2 box-shaped solid.



Three of these solids can be packed into a 2 x 4 x 6 box so that they fill it completely.



Therefore **6 "Els" can be packed in the box.**

53%

5E *Strategy: List the numbers that satisfy each condition. Look for a match.*
The numbers greater than five that leave 1 over when grouped by threes:
7, 10, 13, 16, 19, 22, **25**, 28, …
The numbers greater than five that leave 4 over when grouped by sevens:
11, 18, **25**, 32, …
The smallest number that is on both lists is 25.
The smallest number of marbles Kayla could have is 25.

Follow-Ups: *What is the next smallest number of marbles that Kayla could have?* [46] *What is the third smallest number of marbles that Kayla could have?* [67] *Do you notice a pattern?* [Each number is 21 more than the last.] *Can you explain why the pattern holds?* [To keep both remainders the same while increasing the quotients, the amount of increase must be a multiple of both numbers, i.e. a multiple of their LCM.]

Silver Anniversary Edition 1979–2004

Copyright © 2003 by Mathematical Olympiads for Elementary and Middle Schools, Inc. All rights reserved.