OLYMPIAD PROBLEMS

2007-2008

DIVISION E

WITH

ANSWERS AND SOLUTIONS

MATH OLYMPIADS

Since 1979

Our Twenty-Ninth Year

Mathematical Olympiads for Elementary and Middle Schools

A Nonprofit Public Foundation

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1A ***Time: 3 minutes***

What is the sum?

$$81 + 18 + 72 + 27 + 63 + 36 + 54 + 45 + 4$$

1B ***Time: 5 minutes***

There are a total of 100 men and women on an airplane. There are 12 more women than men. What is the number of women on the airplane?

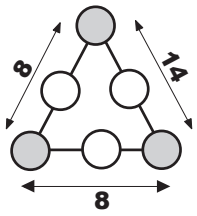
1C ***Time: 5 minutes***

What is the total number of rectangles of ***all*** sizes that can be traced using the lines in this diagram?



1D ***Time: 6 minutes***

Each of the 6 circles contains a different counting number. The sum of the 6 numbers is 21. The sum of the 3 numbers along each side of the triangle is shown in the diagram. What is the sum of the numbers in the shaded circles?



1E ***Time: 6 minutes***

Rebekah's mother is 4 times as old as Rebekah is now. In 6 years, Rebekah's mother will be 3 times as old as Rebekah will be then. What is Rebekah's age now, in years?

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2A ***Time: 3 minutes***

What number makes the statement true?

$$220 - 22 = \underline{\quad\quad} \times 22$$

2B ***Time: 4 minutes***

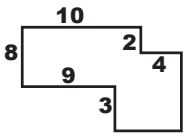
Five students sit around a circular table. Their chairs are numbered in order from 1 through 5. Abby sits next to both Ben and Colin. Dalia sits next to both Ben and Sarah. The numbers on Abby's and Colin's chairs add up to 6. Who sits in chair number 3?

2C ***Time: 5 minutes***

Suppose we call a number *funny* if it is the product of three prime factors, of which exactly two are the same. (For example: 12 = 2 x 2 x 3, so 12 is *funny*.) What is the total number of *funny* numbers between 30 and 60?

2D ***Time: 6 minutes***

In the figure, all angles are right angles and all distances are in meters. What is the area of the figure, in square meters?



2E ***Time: 6 minutes***

The counting numbers are written out as one long string:

1 2 3 4 5 6 7 8 9 1 0 1 1 1 2 …

What is the 100th digit in the string?

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3A ***Time: 3 minutes***

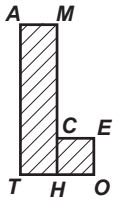
The digits of a four-digit number are 1, 3, 6, and 9, but not necessarily in that order. The thousands digit is prime. The hundreds digit is 3 more than the tens digit. What is the number?

3B ***Time: 5 minutes***

Emma has 4 more quarters than nickels. The total value of her quarters and nickels is \$3.10. In all, what is the number of nickels that Emma has?

3C ***Time: 5 minutes***

The area of rectangle *MATH* is 30 sq cm and each side-length is a counting number of cm. *H* is the midpoint of $\overline{TO}$. The area of square *ECHO* is between 5 sq cm and 24 sq cm. Find the perimeter of the entire figure, in cm.



3D ***Time: 6 minutes***

Sequence *A* counts down from 46 by 7: 46, 39, 32, and so on. Sequence *B* counts down from one number (not 46) by another number (not 7). In sequence *B*, the second number is 35, the sixth number is 23, and the sequence ends with the first single-digit number that it comes to. How many numbers are in sequence *B*?

3E ***Time: 7 minutes***

Amy, Brett, and Cate each secretly write down **Z**, **U**, or **T**. What is the probability that Cate's letter is different from both Amy's letter and Brett's letter?

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4A ***Time: 3 minutes***

January 1, 1990 was a Monday. What day of the week was February 1, 1990?

4B ***Time: 4 minutes***

In an election, Ethan got 5 fewer votes than Christopher, who got 3 votes more than Olivia, who got 4 fewer votes than Ava. How many more votes did Ava get than Ethan?

4C ***Time: 6 minutes***

Mrs. Allen spends $\frac{3}{5}$ of her money at the grocery store. Next she spends $\frac{3}{5}$ of her remaining money at the gas station. She then has $8.00 left. With what total number of dollars did Mrs. Allen start?

4D ***Time: 7 minutes***

Jan and Nika ride their bikes. Jan rides at 5 miles per hour for 1 hour and then rides at 10 miles per hour for 30 minutes. Nika rides at a constant 8 miles per hour. The trips cover the same distance. For how many minutes does Nika ride?

4E ***Time: 6 minutes***

In the multiplication shown, different letters represent different digits. What three-digit number does SAY represent?

$$\begin{array}{r} \text{S A Y} \\ \times \quad\quad 3 \\ \hline \text{B A B Y} \end{array}$$

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5A ***Time: 4 minutes***

A total of 20 marbles are placed into 5 cups. Each cup has a different number of marbles. No cup has exactly 4 marbles, and no cup is empty. What is the greatest number of marbles that any one cup can have?

5B ***Time: 5 minutes***

Aidan writes the counting numbers in order. In Row 1, he writes the first number. In Row 2 he writes the next two numbers, and so on as shown. What is the thirteenth number in Row 16?

Row 1: 1
Row 2: 2 3
Row 3: 4 5 6
... and so on

5C ***Time: 6 minutes***

Joshua has more than 250 toy soldiers. When he tries to arrange them in rows of 3, there are 2 left over. When he tries to arrange them in rows of 5, there are 2 left over. When he tries to arrange them in rows of 7, there are 2 left over. What is the least number of toy soldiers Joshua may have?

5D ***Time: 7 minutes***

The figure shows a regular hexagon: all 6 sides are congruent to each other and all 6 angles are congruent to each other. The area of the shaded rectangular region is 60 sq cm. What is the area of the hexagon, in sq cm? (Hint: Draw the diagonals.)



5E ***Time: 5 minutes***

In the sequence below, we add any two consecutive entries to get the very next entry. The last two entries are 37 and 60 in that order. The first entry is 4. How many entries are in this sequence in all?

4 , ... , ___, 37 , 60 .

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ANSWERS AND SOLUTIONS

Note: For each problem the given per cent indicates the per cent of all competitors with a correct answer.

OLYMPIAD 1 — NOVEMBER 20, 2007

Answers: [1A] **400** [1B] **56** [1C] **24** [1D] **9** [1E] **12**

78% correct

1A *Strategy: Use the pattern in the numbers to simplify the arithmetic.*
(81 + 18) + (72 + 27) + (63 + 36) + (54 + 45) + 4 = 99 + 99 + 99 + 99 + 4 = 4 × 99 + 4 × 1.
Rewrite as 4 × (99 + 1) = 4 × 100 = 400. **The sum is 400.**

FOLLOW–UPS: *All possible 4-digit numbers are formed using the digits 1, 2, 3, and 4. Each digit is used once in each number. (1) What is the sum of all these numbers?* [66,660] *(2) The digits of each 4-digit number are added. What is the sum of all these sums?* [240]

41%

1B **METHOD 1:** *Strategy: Pair each man with a woman.*
There are 12 women who can't be paired with men, and so 100 – 12 = 88 people can be paired. Then 88 ÷ 2 = 44 of the people are men, and 44 + 12 = **56 women are on the plane.**

METHOD 2: *Strategy: Start with equal numbers of men and women.*
Suppose the plane has 50 men and 50 women. To get 12 more women than men, replace 6 men by 6 women. This results in 6 fewer men and 6 more women. Then 44 men and 56 women are on the plane.

METHOD 3: *Strategy: Use algebra (for those familiar with algebra).*
Let m be the number of men. Then m + 12 is the number of women.
(m) +(m + 12) = 100.
Solving, 2m + 12 = 100, 2m = 88, and m = 44.
There are 44 men and 100 – 44 = 56 women on the airplane.

FOLLOW–UP: *There are a total of 60 men and women in a club. If the number of men is two-thirds of the number of women, find the number of women in the club.* [36].

9%

1C **METHOD 1:** *Strategy: Count in an organized way.*
The table organizes each type of rectangle in the diagram by the number of regions and the number of times each occurs.

# of regions	1	2		3		4	6
# of rectangles	8	4	5	2	2	2	1

There are 8 + 4 + 5 + 2 + 2 + 2 + 1 = **24 rectangles of all sizes in the picture.**

METHOD 2: *Strategy: Label each region and list in an organized way.*

A		F
B	D	G
C	E	H

1 bit: **A, B, C, D, E, F, G, H**
2 bits: **AB, BC, DE, FG, GH; BD, DG, CE, EH**

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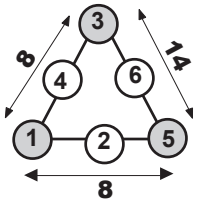
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3 bits: **ABC, FGH; BDG, CEH**
4 bits: **BDCE, DGEH**
6 bits: **BDGCEH.** There are 24 rectangles.

32%

1D **METHOD 2:** *Strategy: Find the sum of the numbers along the three sides.*
8 + 8 + 14 = 30. This sum includes the numbers in all six circles. However, each number in the three shaded circles was counted *twice*. **The sum of the numbers in the three shaded circles is** 30 – 21 = **9.** Note: we do not need to place the numbers in circles.

METHOD 1: *Strategy: Determine the numbers and then their placement.*
The smallest possible sum for six different counting numbers is 1 + 2 + 3 + 4 + 5 + 6 = 21. The sum is given as 21, so the numbers are 1, 2, 3, 4, 5, and 6. Two given sums are 8 and the only sets of these numbers with that sum are {1, 3, 4} and {1, 2, 5}. These numbers are in the five circles along the left and bottom sides of the triangle, and the number they share, 1, is in the bottom left shaded circle. 6 must be in the unshaded circle in the center of side 14. The other two shaded circles on side 14 sum to 8, and can only be 3 and 5, in either order. The three shaded circles contain the numbers 1, 3, and 5, and **their sum is 9.**



33%

1E **METHOD 1:** *Strategy: Build a table using multiples of 6.*
The fact that the mother's age is a multiple of Rebekah's age both now and in 6 years suggests that each age is originally a multiple of 6.

AGE NOW	Rebekah	6	**12**	18
	Mother (4 times as much)	24	**48**	72
AGE IN 6 YEARS	Rebekah	12	**18**	24
	Mother	30	**54**	78
Is Mother 3 times Rebekah's age?		No	**Yes**	No

Rebekah is 12 years old now.

METHOD 2: *Strategy: Start with a simpler problem.*
Instead of 6 years, suppose 1 year elapses. The first two columns show some possible related ages now for Rebekah and her mother. The next two columns show their resulting ages one year from now. The last column, setting aside common sense for the moment, checks if her mother will then be 3 times as old as Rebekah.

Ages now		Ages in 1 year		Is Mom's age
Rebekah	**her mother**	**Rebekah**	**her mother**	**3 times R's**
1	4	2	5	No
2	**8**	**3**	**9**	**Yes**

To increase the elapsed time from 1 year to 6, while preserving the ratios of the ages (i. e. Mother is now 4 times as old and will be 3 times as old), multiply each age by 6. If Rebekah is now 12 and her mother is 48, then in 6 years Rebekah will be 18 and her mother will be 54, which is 3 times as old. Rebekah now is 12.

Follow–Ups: (1) Chandra is twice as old as Nora. Nora is 6 years younger than Lian. The sum of their ages is 62. How old is Nora? [14]. *(2) The length of a rectangle is 5 times its width. Its length and width are both increased by 3 cm. The length of the new rectangle is 4 times its width. Find the perimeter of the original rectangle.* [108 cm]

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OLYMPIAD 2 — DECEMBER 11, 2007

Answers: [2A] **9** [2B] **Dalia** [2C] **4** [2D] **119** [2E] **5**

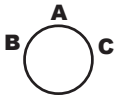
67% correct

2A **METHOD 1:** *Strategy: Simplify the problem.*
Since 220 = 10 × 22, then 220 – 22 = 9 × 22. **9 makes the statement true.**

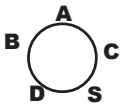
METHOD 2: *Strategy: Do the indicated arithmetic operations.*
220 – 22 = 198 and 198 ÷ 22 = 9. 9 makes the statement true.

68%

2B *Strategy: First determine the order in which the students sit.*
Abby sits next to both Ben and Colin.



Dalia sits next to both Ben and Sara.



Thus Abby and Colin are seated next to each other. Because the chair numbers are in numerical order and theirs add up to 6, Abby and Colin are seated in chairs 1 and 5 in either order. In both cases, **Dalia is in chair number 3.**

FOLLOW-UPS: *Suppose the chairs are not numbered. In how many ways can 5 people be arranged at a circular table? 6 people? 7?* [24; 120; 720]

12%

2C **METHOD 1:** *Strategy: Consider possible values of the repeated prime factor.*
The following are organized according to the repeated factor. Products between 30 and 60 are **underlined and bold**.

2×2×3, 2×2×5, 2×2×7, **2×2×11**, **2×2×13**, 2×2×17, …
3×3×2, **3×3×5**, 3×3×7, …
5×5×2, 5×5×3, …
7×7×2, … No other funny numbers between 30 and 60 are possible.

In all, there are 4 funny numbers, 44, 45, 50, and 52, **between 30 and 60.**

METHOD 2: *Strategy: Factor all the numbers between 30 and 60.*
Factor 30, 31, 32, 33, …, 60. Only four numbers turn out to be funny:

44 = 2 × 2 × 11,
45 = 3 × 3 × 5,
50 = 2 × 5 × 5,
52 = 2 × 2 × 13.

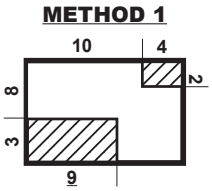
FOLLOW–UPS: *(1) How many different divisors does each funny number have?* [6]
(2) What number between 30 and 60 has this same number of divisors but is not funny? [32]
(3) What is the next smallest number that has exactly 6 divisors but is not funny? [243]

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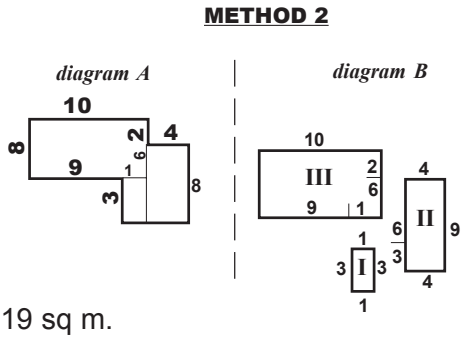
11%

2D *Strategy: Express the desired area in terms of the areas of simpler figures.*

METHOD 1: *Strategy: Embed the given figure in a larger rectangle.*
Wrap the figure in a rectangle, as shown. The rectangle's side-lengths are 14 m and 11 m, and its area is 154 sq m. The areas of the shaded rectangles are 9 × 3 = 27 sq m and 2 × 4 = 8 sq m. Subtracting 27 and 8 from 154, **the area of the given figure is 119 sq m.**



METHOD 2: *Strategy: Split the figure into smaller rectangles.*
One possibility is shown in the diagram.



Opposite sides of a rectangle have the same length. This produces segments of 1 cm and 6 cm as shown in diagram A. Then diagram B shows the separation of the rectangles and the lengths of all sides. Then:

- Rectangle I: Its area is 1 × 3 = 3 sq m.
- Rectangle II: Its area is 9 × 4 = 36 sq m.
- Rectangle III: Its area is 8 × 10 = 80 sq m.

The area of the given figure is then 80 + 3 + 36 = 119 sq m.

42%

2E **METHOD 1:** *Strategy: Count the number of digits in an organized way.*
Count the digits from 1 through 9, then 10 through 19, and so on.

Counting numbers	1⇒9	10⇒19	20⇒29	30⇒39	40⇒49
Number of digits	9	20	20	20	20
Cumulative total number of digits	9	29	49	69	89

The numbers 1 through 49 require 89 digits. The remaining 100 – 89 = 11 digits form the numbers 50, 51, 52, 53, 54, and the first digit of 55. **The 100th digit in the string is 5.**

METHOD 2: *Strategy: Look for a pattern.*
Write the first 100 digits of the string in a 10 by 10 array. After the first 9 numbers, all the remaining numbers are two-digit, requiring two boxes. Each decade (the 10s, 20s, etc.) has twenty digits and takes exactly two rows of the array to complete, starting in the tenth column. Then the digits in the tenth column, read top to bottom, are two 1's, then two 2's, and so on. The digit in the tenth row and tenth column, which is the 100th digit, is a 5.

1	2	3	4	5	6	7	8	9	1
0	1	1	1	2	1	3	1	4	1
5	1	6	1	7	1	8	1	9	2
0	2	1	2	2	2	3	2	4	2
5	2	6	2	7	2	8	2	9	3
0	3	1	3	2	3	3	3	4	3
5	3	6	3	7	3	8	3	9	4
0	4	1	4	2	4	3	4	4	4
5	4	6	4	7	4	8	4	9	5
0	5	1	5	2	5	3	5	4	5

Follow–Ups: *(1) Look at the shaded column of the table shown in method 2. The first zero to appear in this column is a digit of what number?* [103]

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OLYMPIAD 3 — JANUARY 15, 2008

Answers: [3A] **3961** [3B] **7** [3C] **32** [3D] **11** [3E] $\frac{4}{9}$ *or* $\frac{12}{27}$

56% correct

3A *Strategy: Fulfill one requirement at a time.*

3 is the only prime number given .....3 _ _ _
9 is 3 more than 6.....3 9 6 _
1 is the only digit left.....3 9 6 1
The number is 3,961.

(1 is not prime. See *What Every Young Mathlete Should Know.*)

59%

3B **METHOD 1:** *Strategy: Pair each nickel with a quarter.*
After the nickels are paired with quarters, there are 4 quarters left over, worth a total of $1.00. The paired coins are worth $3.10 – $1.00 = $2.10. Each pair is worth $.05 + $.25 = $.30. The number of pairs is 210 ÷ 30 = 7, so **Emma has 7 nickels.**

METHOD 2: *Strategy: Make a table. Find a pattern.*

Number of nickels	1	2	3	…	?
Number of quarters	5	6	7	…	
Value of nickels	$0.05	$0.10	$0.15	…	
Value of quarters	$1.25	$1.50	$1.75	…	
Total value	$1.30	$1.60	$1.90	…	$3.10

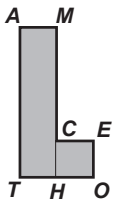
Each time a nickel is added, a quarter is also added and this increases the total value by $.30. When there is 1 nickel, the total value is $1.30. Emma has $3.10 which is $1.80 more than $1.30. Then 180 ÷ 30 is 6. After the first nickel, 6 more are needed. Emma has 7 nickels.

METHOD 3: *Strategy: Work down from the maximum number of quarters.*
Emma can't have more than $3.00 in quarters. Suppose she has 12 quarters. Then she would have 8 nickels, for a total value of $3.40, which is too high. If, instead, she has 11 quarters, she would have 7 nickels for a total value of $3.10. Emma has 7 nickels.

FOLLOW-UP: *Cary's bank contains quarters, nickels and dimes. The number of nickels is 3 times the number of quarters. The number of dimes is 3 times the number of nickels. The bank has 39 coins in all. What is the total value of Cary's coins?* [$3.90]

15%

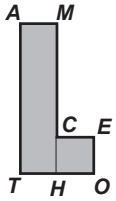
3C **METHOD 1:** *Strategy: Start with the area of MATH.*
Rectangle *MATH* is either 1 by 30, 2 by 15, 3 by 10, or 5 by 6 cm.



Since *TH* = *HO*, the area of square *ECHO* must be 3^2 or 9, since the squares of 1, 2, 5, 6, 10, 15, and 30 are not between 5 and 24. Then *TH* = 3, *HO* = 3, *OE* = 3, *EC* = 3, *CH* = 3, *AM* = 3, *AT* = 10. Then *MH* = 10, so *MC* = 7.

The perimeter of the shaded figure, *MA* + *AT* + *TH* + *HO* + *OE* + *EC* + *CM*, **is 32 cm.**

METHOD 2: *Strategy: Start with the area of ECHO.*
The area of *ECHO* could be 9 or 16 sq cm, so *HO* could be 3 or 4 cm. Since *H* is the midpoint of $\overline{TO}$, *TH* is either 3 or 4 cm.



The area of *MATH* is 30 sq cm. Of 3 and 4, only 3 is a factor of 30, so each side of *ECHO* is 3 cm long.

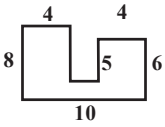
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Thus, *TH* = 3 cm and *AT* = 10 cm. Further, *MA* = 3 and *MC* = 10 – 3 = 7 cm.

Then the perimeter of the shaded figure is 32 cm, the sum of its sides.

FOLLOW-UP: All measurements in the figure are in cm and all angles are right angles. (1) What is the perimeter of the figure? [46 cm] (2) What is its area? [58 sq cm]



43%

3D *Strategy: Find the common difference (the number by which Sequence B counts down).*
Represent Sequence B visually: __, 35, __, __, __, 23, __, __, … .

The sequence counts down from 35 to 23 in 4 steps, so each step counts down by 3. Keep counting down by 3 until a single-digit number is reached. Sequence B is 38, 35, 32, 29, 26, 23, 20, 17, 14, 11, 8. **There are 11 numbers in Sequence B.**

4%

3E **METHOD 1:** *Strategy: Compare Cate's letter with each of the others, one at a time.*
For each letter Cate might choose, the probability that Amy chooses a different letter is $\frac{2}{3}$. In $\frac{1}{3}$ of those cases Brett's letter will match Cate's letter, and thus in $\frac{2}{3}$ of the cases Brett's letter will differ from Cate's letter. That is, Brett's letter will differ from Cate's letter $\frac{2}{3}$ of $\frac{2}{3}$ of the time. Therefore, **the probability that Cate's letter is different from both Amy's and Brett's is $\frac{4}{9}$.**

METHOD 2: *Strategy: Find all instances in the sample space that meet the conditions given.*
Each has a choice of 3 letters. Amy and Brett could have the same letter. The sample space has 3 × 3 × 3 = 27 possibilities. Write them out as a chart as follows or as a tree diagram. The instances when Cate's letter is different from both Amy's letter and Brett's letter are highlighted.

Cate chooses Z	ZZZ	ZZU	ZZT	ZUZ	**ZUU**	**ZUT**	ZTZ	**ZTU**	**ZTT**
Cate chooses U	**UZZ**	UZU	**UZT**	UUZ	UUU	UUT	**UTZ**	UTU	**UTT**
Cate chooses T	**TZZ**	**TZU**	TZT	**TUZ**	**TUU**	TUT	TTZ	TTU	TTT

In each case Cate's number is listed first.

The probability that Cate's letter is different from both Amy's and Brett's is $\frac{12}{27}$ or $\frac{4}{9}$.

FOLLOW-UP: Amy, Brett, Cate, and David each write down 1, 2, 3, or 4. What is the probability that each writes down a different number? [$\frac{3}{32}$]

OLYMPIAD 4 — FEBRUARY 5, 2008

Answers: [4A] **Thursday** [4B] **6** [4C] **50** [4D] **75** [4E] **570**

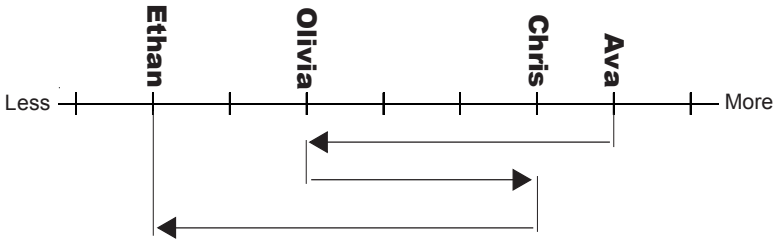
61% correct

4A *Strategy: Find the number of full weeks in January.*
January has 31 days, which is 4 weeks and 3 days. Four weeks after Monday, January 1, is another Monday, January 29. Three days later is a Thursday. **February 1, 1990, was on a Thursday.**

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52%

4B **METHOD 1:** *Strategy: Work backwards, using a number line.*
Start with Ava.



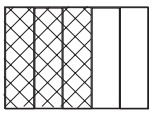
Counting spaces, **Ava got 6 more votes than Ethan.**

METHOD 2: *Strategy: Choose an arbitrary number of votes for Ethan.*
Because the four vote totals differ by fixed amounts, assigning any random value to Ethan's total will not change the difference between his total and Ava's. Suppose Ethan received 10 votes. Then Christopher had 15 votes, Olivia had 12 votes and Ava had 16 votes. Then Ava beat Ethan by 16 – 10 = 6 votes.

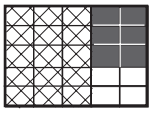
FOLLOW-UP: *Will read six times as many books as Nia, who read one-quarter as many as Jeff, who read twice as many as Paula. Paula read 10 books. Who read more books, Jeff and Paula combined,or Will?* [The sums are equal.]

21%

4C **METHOD 1:** *Strategy: Represent the fractions visually.*
In the first diagram, the cross-hatching represents the $\frac{3}{5}$ of Mrs. Allen's original sum that she spent at the grocery store, so $\frac{2}{5}$ of the starting sum remained, as shown by the unshaded boxes.



Next, the solid shading in the second diagram represents what she spent at the gas station, which is $\frac{3}{5}$ of the first remainder. The 4 unshaded boxes represent her final remainder, \$8. Then each box represents \$2. The original, large, box contains 25 of these small boxes and so represents 25 × 2 = \$50. **Mrs. Allen started with \$50.**



METHOD 2: *Strategy: Find the fractional part of her money that is left.*
After one purchase, she has $\frac{2}{5}$ of her starting sum left. After two purchases, she has $\frac{2}{5}$ of that $\frac{2}{5}$ left, which is $\frac{4}{25}$ of her starting sum. Since $\frac{4}{25}$ of the starting sum is \$8, then $\frac{8}{50}$ of it is \$8, $\frac{1}{50}$ of it is \$1, and she started with \$50.

22%

4D *Strategy: Find the distance each rides.*
At 5 miles per hour for 1 hour, Jan travels 5 miles. Then at 10 miles per hour for a half hour, she travels half of 10, or 5, miles. So Jan travels a total of 5 + 5 = 10 miles.

Then Nika also travels 10 miles. After 1 hour, she has traveled 8 miles and has 2 miles to go. That 2 miles is $\frac{1}{4}$ of the distance she has already traveled and so takes $\frac{1}{4}$ as much time. Nika needs 60 minutes to cover the first 8 miles and an additional 15 minutes to cover the last 2 miles, so **Nika rides for 75 minutes.**

FOLLOW-UP: *What was Jan's average speed, in mph?* [$6\frac{2}{3}$]

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10%

4E *Strategy: List and examine each possibility for **B** and **Y**.*
Since **SAY** is less than 1000, **BABY** is less than 3000 and **B** can only be 2 or 1.
The product of **Y** and 3 has a ones digit of **Y**, so **Y** can only be 5 or 0.
Then there are just four choices to test for (**B**,**Y**): (2,5) or (1,5) or (2,0) or (1,0).

(1) Suppose **B** = 2 and **Y** = 5. Thus 1 more than 3 × **A** ends in 2, so **A** = 7. Then 2 more than 3 × S is 27, which is impossible.

```
  S A 5
×     3
2 A 2 5
```

(2) Suppose **B** = 1 and **Y** = 5. Thus 1 more than 3 × **A** ends in 1, so **A** = 0. Then 3 × **S** is 10. This is also impossible.

```
  S A 5
×     3
1 A 1 5
```

(3) Suppose **B** = 2 and **Y** = 0. Thus 3 × **A** ends in 2, so **A** = 4. Then 1 more than 3 × **S** is 24, which is again impossible.

```
  S A 0
×     3
2 A 2 0
```

(4) The only remaining possibility is **B** = 1 and **Y** = 0. Thus 3 × **A** ends in 1, so **A** = 7. Then 2 more than 3 × **S** is 17, so **S** = 5. Checking, 570 × 3 = 1710.

```
  S A 0
×     3
1 A 1 0
```

The three-digit number represented by SAY is 570.

OLYMPIAD 5 — MARCH 4, 2008

Answers: [5A] **9** [5B] **133** [5C] **317** [5D] **90** [5E] **7**

36% correct

5A *Strategy: Minimize the other numbers.*
To get as many marbles as possible in one cup, place as few as possible in the others. The conditions of the problem show that the least number of marbles in the first four cups is 1 + 2 + 3 + 5 = 11. With a total of 20 marbles, **the greatest number any one cup can have is 9.**

40%

5B *Strategy: Look for a pattern in the last numbers in each row.*
Examine the last number in each row. Figure 1 shows that it is the sum of that row number and all the row numbers preceding it. These numbers (1, 3, 6, 10, …) are called "triangular numbers".

Row	Last Number
1	1
2	3 = 1 + 2
3	6 = 1 + 2 + 3
⋮	⋮
16	1 + 2 + 3 + … + 16

Figure 1

The last number in the 16th row is the 16th triangular number, 1 + 2 + 3 + 4 + … + 15 + 16. To add them quickly, use the symmetry shown in figure 2. Pair from the outside in: add 1 and 16, 2 and 15, 3 and 14, and so on. There are 8 pairs of numbers, each adding to 17, and the total is 8 × 17 = 136.

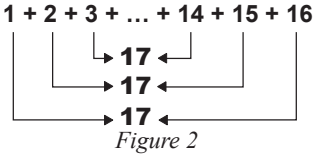


Figure 2

Counting back three numbers, **the 13th number in row 16 is 133.**

FOLLOW-UP: *What is the last number in the 11th row of this pattern?*

Row 1:	*1*			
Row 2:	*3*	*5*		
Row 3:	*7*	*9*	*11*	
Row 4:	*13*	*15*	*17*	*19*

[131]

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14%

5C **METHOD 1:** *Strategy: Use common multiples.*
In each case 2 toy soldiers were left over. Suppose Joshua had 2 fewer toy soldiers. Then their number would be a multiple of 3, of 5, and of 7. No two of these numbers have a common factor, so their least common multiple is their product, 105. The number of soldiers is thus 2 more than a multiple of 105. The least multiple of 105 that is more than 250 is 315, so **the least number of toy soldiers Joshua may have is** 315 + 2 = **317.**

METHOD 2: *Strategy: List the numbers that satisfy each condition.*
Numbers that are 2 more than a multiple of:

7: 254, 261, 268, 275, 282, …, 310, **317**, 324, …
5: 252, 257, 262, 267, 272, 277, 282, …, 307, 312, **317**, 322, …
3: 251, 254, 257, 260, 263, 266, …, 314, **317**, 320, …

The least number of toy soldiers that Joshua may have is the least number that appears in all 3 lists, 317.

Follow-Ups: *(1) What is the least four-digit number divisible by 5, 7, 15, and 21?* [1050] *(2) What is the least number that leaves a remainder of 3 when divided by 5, a remainder of 4 when divided by 6, and a remainder of 5 when divided by 7? (Hint: Find a nearby number that is a multiple of 5, 6, and 7.)* [208]

23%

5D *Strategy: Split the figure into more familiar shapes.*
Draw the three diagonals of the hexagon.

METHOD 1: Figure 1: The three diagonals divide the hexagon into six congruent triangles. Two of the triangles are completely shaded. The other four triangles are half shaded and half clear.


Figure 1

Figure 2: The halves of the partly shaded triangles can be rearranged to form two clear triangles and two additional completely shaded ones. The four shaded triangles have a total area of 60 sq cm, so each the area of each is 15 sq cm. This is also the area of each clear triangle, so **the area of the entire hexagon is** 6 × 15 = **90 sq cm.**


Figure 2

METHOD 2: In addition to the diagonals, draw also a vertical line through the center of the hexagon. These lines split the hexagon into twelve congruent triangles, eight of which are shaded (figure 3). The shaded area is 60 sq cm, so each shaded triangle has an area of $60 \div 8 = 7\frac{1}{2}$ sq cm and the area of the entire hexagon is $7\frac{1}{2} \times 12 = 90$ sq cm.


Figure 3

24%

5E *Strategy: Work backwards.*
The last term, 60, is the sum of the preceding term, 37, and the term before that, 23. Writing the sequence in reverse order and subtracting each term from the preceding one gives 60, 37, 23, 14, 9, 5, 4. **There are 7 entries in the sequence.**

Follow-Up: *Exploration: These numbers are based on the Fibonacci Sequence, which is 1, 1, 2, 3, 5, 8, 13, 34, 55, and so on. Investigate its properties, history (bee or rabbit births), and applications (where in nature this sequence occurs).*