OLYMPIAD PROBLEMS

2008-2009

DIVISION E

WITH
ANSWERS AND SOLUTIONS

MATH
OLYMPIADS
Since 1979

Our Thirtieth Year

Mathematical Olympiads for Elementary and Middle Schools

A Nonprofit Public Foundation

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1A ***Time: 3 minutes***

What is the value of the following?

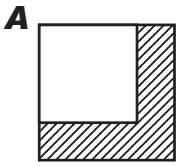
$$55 - 11 + 44 - 22 + 33 - 33 + 22 - 44 + 11 - 55$$

1B ***Time: 5 minutes***

In all, how many two-digit prime numbers have 4 as one of their digits?

1C ***Time: 5 minutes***

In the figure shown, two squares share corner A. The larger square has an area of 49 sq cm. The smaller square has an area of 25 sq cm. What is the perimeter of the shaded region, in cm?



1D ***Time: 5 minutes***

Janine's number has three digits. One digit is a prime number. Another digit is a square number. The other digit is neither prime nor square. Her number is NOT divisible by 3. What is the greatest possible value of Janine's number?

1E ***Time: 6 minutes***

In all, how many whole numbers between 400 and 600 are divisible by 9?

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2A *Time: 4 minutes*

Aaron, Becky, and Chris are wearing pink, yellow, and white shirts (though not necessarily in that order). Aaron is not wearing the yellow shirt. Becky says to Chris, "I like your white shirt." What color is Becky wearing?

2B *Time: 5 minutes*

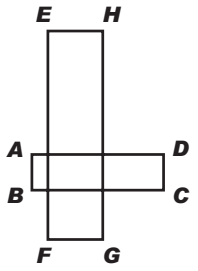
If a three-digit number is divided by 5 or by 6, the remainder is 1 in each case. What is the least such three-digit number?

2C *Time: 5 minutes*

Abby lists four consecutive multiples of some number. The average of the first two multiples is 28 and the average of the last two is 44. What is the greatest multiple on Abby's list?

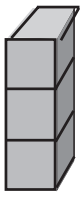
2D *Time: 6 minutes*

As shown, the overlap of rectangles *ABCD* and *EFGH* is also a rectangle. The area of *ABCD* is 14 sq cm. The area of *EFGH* is 33 sq cm. *AD* = 7 cm and *EF* = 11 cm. Find the area of the entire figure, in sq cm.



2E *Time: 6 minutes*

Three identical cubical boxes form a stack. It takes 350 sq cm of wrapping paper to completely wrap the whole stack with no overlap. Suppose each cube is wrapped separately and completely instead. What is the least amount of additional paper that is needed, in sq cm?



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3A ***Time: 4 minutes***

The four-digit number 489A is divisible by 11.
What digit does A represent?

3B ***Time: 5 minutes***

Emma has $1.85 in her pocket, all in quarters and nickels. Her father gives her another quarter. She now has the same number of quarters as nickels. How many nickels does Emma have in all?

3C ***Time: 5 minutes***

Tony has an 8 cm by 12 cm paper rectangle. He folds it in half three times, each time making a smaller rectangle. What is the least possible perimeter of the rectangle after the third fold, in cm?

3D ***Time: 5 minutes***

If 4 people can paint 2 fences in 5 hours, how many hours in all will it take for 8 people to paint 8 fences?

3E ***Time: 7 minutes***

7^2 means 7×7 and its product is 49, which has a units digit of 9.
7^3 means $7 \times 7 \times 7$ and its product is 343, which has a units digit of 3.
7^4 means $7 \times 7 \times 7 \times 7$ and its product is 2401, which has a units digit of 1.
What is the units digit in the product of 7^{50}?

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4A ***Time: 4 minutes***

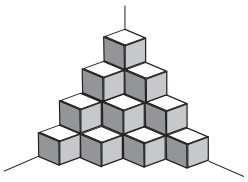
A digital clock shows 2:35. This is the first time after midnight when *all three* digits are different prime numbers. What is the last time before noon when *all three* digits on the clock are different prime numbers?

4B ***Time: 5 minutes***

The only way that 10 can be written as the sum of 4 different counting numbers is 1 + 2 + 3 + 4. In how many different ways can 15 be written as the sum of 4 different counting numbers? Assume that order does not matter.

4C ***Time: 5 minutes***

The tower shown is made of congruent cubes stacked on top of each other. Some of the cubes are not visible. How many cubes in all are used to form the tower?



4D ***Time: 6 minutes***

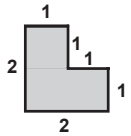
Hannah gives clues about her six-digit secret number:

Clue 1: It is the same number if you read it from right to left.
Clue 2: The number is a multiple of 9.
Clue 3: Cross off the first and last digits. The *only* prime factor of the remaining four-digit number is 11.

What is Hannah's six-digit number?

4E ***Time: 7 minutes***

Zach has 5 tiles, each shaped like the one at the right. Each tile is formed from three 1-cm squares. He moves the 5 tiles together to create a larger figure. What is the least possible perimeter of this larger figure, in cm?



All lengths shown are in cm.

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5A ***Time: 4 minutes***

Sara said, "If you divide my age by 3 and then add 8 years, the result is my age." How old is Sara, in years?

5B ***Time: 5 minutes***

In the list of numbers 5, 8, 11, 14, … , each number is 3 more than the number before it. What is the first number in the list that is greater than 100?

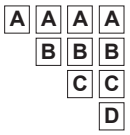
5C ***Time: 7 minutes***

In the following, different letters represent different digits. What digit does the letter O represent?

$$\begin{array}{r} \text{O N E} \\ +\ \text{F O U R} \\ \hline \text{F I V E} \end{array} \qquad\qquad \begin{array}{r} \text{O N E} \\ \times\ \quad 4 \\ \hline \text{F O U R} \end{array}$$

5D ***Time: 7 minutes***

Each of 10 cards displays two different letters (**A**, **B**, **C**, **D**, or **E**), one on the front and the other on the back. The front of each card is shown. No two cards have the same pair of letters. What is the fewest cards you can turn over and be absolutely sure that an **E** will appear?



5E ***Time: 5 minutes***

Suppose A and B represent any two numbers and the value of $A \circledcirc B$ is given by the formula $(A \times A) - (B \times B)$. What is the sum of $(8 \circledcirc 5)$ and $(5 \circledcirc 2)$?

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ANSWERS AND SOLUTIONS

Note: Number in parentheses indicates percent of all competitors with a correct answer.

OLYMPIAD 1 — NOVEMBER 18, 2008

Answers: [1A] **0** [1B] **3** [1C] **28** [1D] **985** [1E] **22**

84% correct

1A *Strategy: Rearrange the numbers to simplify the arithmetic.*
55 – 11 + 44 – 22 + 33 – 33 + 22 – 44 + 11 – 55 =
(55 – 55) + (44 – 44) + (33 – 33) + (22 – 22) + (11 – 11) = 0
The value is 0.

FOLLOW–UP: *What is the value of the following?*
$(1 - \frac{1}{2}) + (\frac{1}{2} - \frac{1}{3}) + (\frac{1}{3} - \frac{1}{4}) + (\frac{1}{4} - \frac{1}{5}) + (\frac{1}{5} - \frac{1}{6}) + (\frac{1}{6} - \frac{1}{7})$ [ans. $\frac{6}{7}$]

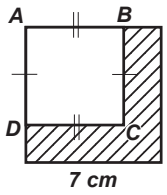
38%

1B *Strategy: Consider the ones and tens digits separately*
Any number with a ones digit of 4 is a multiple of 2 and therefore not prime. If the tens digit is 4, the number could be 41, 43, 45, 47, or 49. But 45 = 9 × 5, and 49 = 7 × 7. The others are prime. **There are 3 two-digit prime numbers that have a digit of 4.**

FOLLOW-UP: The sum of two prime numbers is 63. Find their product. [122]

21%

1C **METHOD 1:** *Strategy: Find a simpler equivalent problem.*
The area of the large square is 49 sq cm, so each side is 7 cm. Since the sides of square ABCD are congruent, DC = AB and BC = AD. **The perimeter of the shaded region is** then the same as the perimeter of the large square, 4 × 7 = **28 cm.** (*Note that the size of ABCD is irrelevant!*)



METHOD 2: *Strategy: Find the dimensions of the shaded area.*
Since the area of the large square is 49 sq cm, each side is 7 cm. The area of the small square is 25 sq cm, and each side is 5 cm. The difference between the larger and the smaller square is 2 cm. Therefore the sides of the shaded region are 7, 7, 5, 5, 2, and 2 cm and its perimeter is 28 cm.

7%

1D *Strategy: Determine which digits satisfy each condition.*
The single digit prime numbers are 2, 3, 5, and 7. The single digit perfect squares are 0, 1, 4, and 9, leaving 6 and 8 as neither prime nor perfect square. Choose the largest digit in each group and arrange from greatest to least to get the largest possible number, 987. But 987 is divisible by 3. To get the next greatest number, take the next greatest digit from the group containing 7. **The greatest possible value of Janine's number is 985.**

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27%

1E **METHOD 1:** *Strategy: Find the number of multiples of 9 less than each given value.*
Since $600 \div 9 = 66\frac{2}{3}$, there are 66 numbers less than 600 that are divisible by 9. Since $400 \div 9 = 44\frac{4}{9}$, there are 44 numbers less than 400 that are divisible by 9. Because neither 400 nor 600 is divisible by 9, **there are** 66 – 44 = **22 numbers between 400 and 600 that are divisible by 9.**

METHOD 2: *Strategy: Split the numbers in the interval into groups of 9.*
Between 400 and 600, there are 199 whole numbers, excluding 400 and 600. Dividing 199 by 9 yields 22 with a remainder of 1. Thus there are 22 complete groups of 9 consecutive numbers. Each group contains exactly one multiple of 9. The "leftover" number, 599, is not a multiple of 9, so there are 22 numbers in the interval that are divisible by 9.

METHOD 3: *Strategy: Find the first number and repeatedly add 9.*
$400 \div 9 = 44\frac{4}{9}$, so the first number greater than 400 that is divisible by 9 is 45 × 9 = 405. Then 405, 414, 423, 432, 441, 450, 459, 468, 477, 486, 495, 504, 513, 522, 531, 540, 549, 558, 567, 576, 585, and 594 are all divisible by 9, 22 numbers in all.

FOLLOW-UPS: How many numbers between 401 and 599 are divisible (1) by 2 and 5? (2) by 2 or 5? [19; 119]

OLYMPIAD 2 — DECEMBER 16, 2008

Answers: [2A] **Yellow** [2B] **121** [2C] **48** [2D] **41** [2E] **100**

90% correct

2A *Strategy: Use reasoning.*
Aaron is not wearing yellow and Chris is wearing white, so Aaron is wearing pink and **Becky's shirt is yellow.**

37%

2B **METHOD 1:** *Strategy: Use the least common multiple.*
The number sought is 1 more than a multiple of 5 and also 1 more than a multiple of 6. Since 5 and 6 have no factors in common, their least common multiple is 5 × 6 = 30. The smallest three-digit multiple of 30 is 120, so **the least such number is 121.**

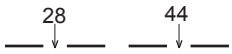
METHOD 2: *Strategy: Make two lists and look for a common member.*
Three-digit multiples of 5: 100, 105, 110, 115, 120, …
Three-digit multiples of 6: 102, 108, 114, 120, …
The least number to leave a remainder of 1 when divided by 5 or by 6 is 121.

FOLLOW-UP: What is the smallest number greater than 1 which leaves a remainder of 1 when divided by 2,3,4,5,6,7,8, or 9? [2521]

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13%

2C **METHOD 1:** *Strategy: Find the interval between consecutive multiples.*
Draw four blanks to represent the four consecutive multiples. 28 is midway between the first two multiples and 44 is midway between the last two. It takes two equal "jumps" in value to go from 28 to 44, so each jump is 8. It takes half a jump, 4, to go from 44 to the greatest multiple, so **the greatest multiple on Abby's list is 48.**



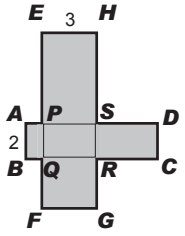
METHOD 2: *Strategy: Examine number pairs that average 28.*
Number pairs that average 28: (27,29), (26,30), (25,31), (24,32), and so on.
Of these pairs, only 24 and 32 are consecutive multiples of a number, namely 8. The next two multiples of 8 are (40,48), which do average 44. The greatest multiple is 48.

METHOD 3: *Strategy: Find the number of which each is a multiple.*
The sum of the first two multiples is 28 × 2 = 56. The sum of the last two multiples is 44 × 2 = 88. If the list of multiples is extended, both sums would also appear on the list. The greatest common factor of 56 and 88 is 8, so that the numbers on the list are 24, 32, 40, and 48, of which 48 is greatest.

FOLLOW-UPS: (1) On a list of ten numbers, the average of the first 2 is 20, the average of the next 3 is 30, and the average of the last 5 is 50. What is the average of all ten numbers? [38]
(2) List any four consecutive multiples of your favorite number. What is the difference between the sum of the last two and the sum of the first two? [4 times your favorite number]
How could this have been used in the solution of the problem?

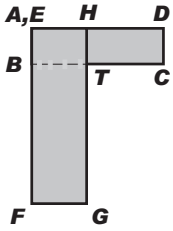
18%

2D **METHOD 1:** *Strategy: X*
The area of *ABCD* is 14 sq cm and *AD* = 7 cm, so *AB* = 14 ÷ 7 = 2 cm. *PQ* = *AB*, so *PQ* = 2 cm. Likewise, *PS* = *EH* = 33 ÷ 11 = 3 cm.



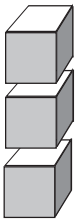
METHOD 1: *Strategy: Find the dimensions of the overlap (PQRS).*
The sum of the areas of rectangles *ABCD* and *EFGH* is 47 sq cm. The overlapping rectangle is part of both rectangles *ABCD* and *EFGH,* and thus its area of 6 sq cm has been included twice in the total. **The area of the shaded region is** 47 – 6 = **41 sq cm.**

METHOD 2: *Strategy: Draw a simpler diagram.*
Move rectangle *ABCD* so that *A* coincides with *E* as shown. As in Method 1, *AB* = 2 cm and *EH* = 3 cm. Then *HD* = 7 – 3 = 4 cm. The area of the shaded region = the area of *EFGH* + the area of *HTCD* = 33 + (4 × 2) = 41 sq cm.



11%

2E. *Strategy: Find the area of one face of a cube.*
The surface of the stack consists of 14 squares, the top and bottom of the stack and 3 squares on each of the four sides of the stack. The area of each square is 350 ÷ 14 = 25 sq cm. If the cubes are wrapped separately, each cube has 6 square faces to cover, a total of 3 × 6 = 18 faces. That is 4 more faces than were covered originally and therefore 4 × 25 = **100 sq cm of additional paper is needed.**



FOLLOW-UP: Five cubes, whose edges measure 1,2,3,4, and 5 cm, respectively, are stacked

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so that each of them except the largest rests on top of the next largest one. How many sq cm of paint are needed to cover the exposed surface of the pile? (Include the bottom face of the largest cube.) [270]

OLYMPIAD 3 — JANUARY 13, 2009

Answers: [3A] **5** [3B] **7** [3C] **14** [3D] **10** [3E] **9**

66% correct

3A **METHOD 1:** *Strategy: Use the test for divisibility by 11.*
If a number is divisible by 11, the difference of the sums of alternating digits is either 0 or some other multiple of 11. (4 + 9) – (8 + A) is divisible by 11 if **A represents the digit 5.**

METHOD 2: *Strategy: Do the division.*

```
   44?
11)489A
   44
   49
   44
   5A
   55
    0
```

If the 2-digit number 5A is divisible by 11, A represents 5.

FOLLOW-UP: (1) What digit might A represent if 87A4 is divisible by 8? [1, 5, or 9]. *(2) Find all values for A and B for which the five digit number 16A3B is divisible by 44.* [There are 2 solutions: (A,B) = (6,2) or (2,6).]

70%

3B **METHOD 1:** *Strategy: Equalize the number of each type of coin.*
After she gets an additional quarter, Emma has $1.85 + $.25 = $2.10. Pair each nickel with a quarter. Each pair is worth $.30, so there are 210¢ ÷ 30¢ pairs. **Emma has 7 nickels.**

Instead of adding a quarter, you could just subtract a nickel, and then follow the above reasoning.

METHOD 2: *Strategy: Consider the **maximum** number of quarters **at first.***
Since Emma began with $1.85, she began with fewer than 8 quarters. The table uses the number of quarters and the corresponding number of nickels to find the total value.

# of quarters to start	7	**6**	5
# of nickels to start	8	**7**	6
Value of these coins	$2.15	**$1.85**	$1.55

If Emma has 6 quarters and 7 nickels, the conditions of the problem are satisfied. Emma has 7 nickels.

FOLLOW-UP: A farm has 8 more chickens than cows. The cows and chickens have a total of 166 legs. How many chickens are on the farm? [33]

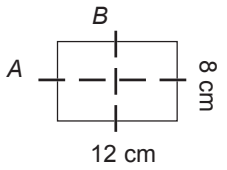
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19%

3C **METHOD 1:** *Strategy: Strategy: Minimize the perimeter after each fold.*
To reduce the perimeter as much as possible at each step, fold the longer side in half.
Start with an 8 cm by 12 cm rectangle. After one fold it is 8 cm by 6 cm. After two folds it is 4 cm by 6 cm. After 3 folds it is 4 cm by 3 cm. **The least possible perimeter after 3 folds is** 2 × 4 + 2 × 3 = **14 cm.**

METHOD 2: *Strategy: Examine all the possibilities.*
Denote by *A* a fold that halves the vertical side and by *B* a fold that halves the horizontal side. Note that the number of folds in each direction but not the order of the folds

Folds	*AAA*	*AAB*	*ABB*	*BBB*
Dimensions after 1 fold	12 × 4	12 × 4	12 × 4	6 × 8
… after 2 folds	12 × 2	12 × 2	6 × 4	3 × 8
… after 3 folds	12 × 1	6 × 2	3 × 4	1.5 × 8
Final perimeter	**26 cm**	**16 cm**	**14 cm**	**19 cm**



The least possible perimeter after 3 folds is 14 cm.

40%

3D **METHOD 1:** *Strategy: First adjust the number of fences.*
Two fences can be painted by 4 people in 5 hours, so 8 fences takes those same 4 people 4 times as long, or 20 hours. If 4 people need 20 hours to paint the 8 fences, then 8 people will take only half as long to do the job. **It takes 10 hours for 8 people to paint 8 fences.**

METHOD 2: *Strategy: First adjust the number of people.*
If 4 people can paint 2 fences in 5 hours, it takes 8 people only half as long ($2\frac{1}{2}$ hours) to paint the 2 fences. To paint 8 fences, the 8 people need $4 \times 2\frac{1}{2} = 10$ hours.

METHOD 3: *Strategy: Find the number of "person-hours" needed to paint a fence.*
If 4 people can paint 2 fences in 5 hours, then one person takes 4 times as long, 20 hours, to paint 2 fences. Then one person can paint 1 fence in 10 hours. Thus 8 people can paint 8 times as many fences, 8 fences, in the same 10 hours.

13%

3E **METHOD 1:** *Strategy: Start with simpler cases and look for a pattern.*
To form the sequence of units digits start with 7, 9, 3, and 1. Then repeatedly multiply the units digit of the last term by 7 and discard the tens digit to find each succeeding term. The first eight terms are 7, 9, 3, 1, 7, 9, 3, 1, and a pattern appears.

The product of four factors of 7 has a ones digit of 1, as does the product of 8, 12, 16 factors of 7, and so on. Thus, every fourth term is 1. The 48^{th} term, which is the ones digit of the product of 48 factors of 7, is also 1. Since 1 × 7 ×7 ends in 9, the 50^{th} term, which is **the ones digit of the product of 50 factors of 7, is 9.**

METHOD 2: *Strategy: Find an equivalent but simpler problem.*
Group the 50 factors of 7 into 25 pairs: (7 × 7) × (7 × 7) × ⋯ × (7 × 7). The ones digit in each of the products is 9, so the ones digit in the product of 50 factors of 7 is the same as the ones digit in the product of 25 factors of 9. But the product of an even number of 9s ends in 1, and the product of an odd number of 9s ends in 9. The ones digit in the product of 25 factors of 9 is 9.

FOLLOW-UPS: (1) Repeated multiplication by 7 produces 4 different ones digits: 7,9,3,1. What other numbers when repeatedly multiplied also produce 4 different ones digits? [2,3,8]
(2) What is the tens digit in the product of fifty factors of 7? [4]

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OLYMPIAD 4 — FEBRUARY 3, 2009

Answers: [4A] **7:53** [4B] **6** [4C] **20** [4D] **513 315** [4E] **16**

33% correct

4A *Strategy: List the single-digit primes.*

The single-digit prime numbers are 2, 3, 5, and 7. Select the 3 greatest numbers from this list and write them from largest to smallest. **The last time before noon when all 3 digits are prime is 7:53.**

FOLLOW-UP: *At how many times between midnight and noon will the digits be 3 different primes?* [18]

13%

4B *Strategy: Make an organized list.*

List all the ways four different numbers can add to 15. Starting with the largest number reduces the number of trials necessary. Because 3 + 2 + 1 = 6 is the least possible sum for 3 of the numbers, the greatest number can't exceed 15 – 6 = 9.

15 = 9 + 6 = 9 + 3 + 2 + 1
15 = 8 + 7 = 8 + 4 + 2 + 1
15 = 7 + 8 = 7 + 5 + 2 + 1 or 7 + 4 + 3 + 1
15 = 6 + 9 = 6 + 5 + 3 + 1 or 6 + 4 + 3 + 2

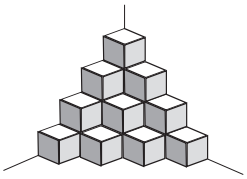
15 can be written as the sum of four different counting numbers in 6 different ways.

46%

4C **METHOD 1:** *Strategy: Count horizontally, from the top layer down.*

Make a table that counts cubes separately for each layer. In each case, add the number of hidden cubes to the number of visible cubes.

Layer	Cubes in layer (visible + hidden)
1 (top)	1 = 1
2	2 + 1 = 3
3	3 + 3 = 6
4 (bottom)	4 + 6 = 10



1 + 3 + 6 + 10 = **20 cubes are used to form the tower.**

METHOD 2: *Strategy: Count vertically, stack by stack.*

This table counts cubes separately for each stack (column), from the shortest to the tallest. Both hidden and visible cubes are counted.

Height of stack	No. of stacks	Total cubes by height
1	4	4
2	3	6
3	2	6
4	1	4

20 cubes are used to form the tower.

FOLLOW-UPS: (1) How many cubes in all would be in a similar tower of 5 layers? 6? 7? [35, 56, 84] *(2) Suppose the bottom layer of a similar tower has 91 cubes. How many layers would there be?* [13]

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4%

4D *Strategy: Consider the clues one at a time, starting with the most restrictive.*

Clue 3: The only prime factor of the 4-digit number is 11, so the number = 11×11 or 11×11×11 or 11×11×11×11, etc. Of these, only 11×11×11= 1331 has 4 digits, so the middle 4 digits are 1331.

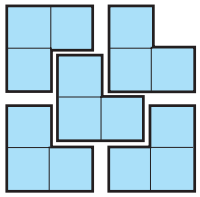
Clue 1: The number reads the same right to left, so the first and last digits are the same. Call the number A1331A.

Clue 2: The number is a multiple of 9, so the sum of its digits is a multiple of 9. A + 1 + 3 + 3 + 1 + A = A + 8 + A must equal 9 or 18. No digit A satisfies A + 8 + A = 9, but if A + 8 + A = 18, A = 5. **Hannah's number is 513 315.**

8%

4E *Strategy: Make the figure as compact as possible.*

The area of the given L-shape is 3 sq cm. A figure made up of 5 L-shapes has an area of 15 sq cm. The fact that all the angles in each shape are right angles suggests trying to pack the 5 shapes into a square of area 16 sq cm.This can be done as shown in the figure. **The least possible perimeter is** 4 + 4 + 3 + 3 + 1 + 1 = **16 cm.**



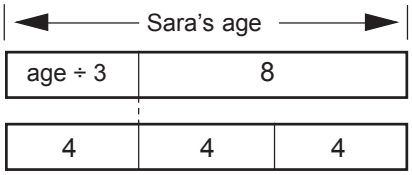
OLYMPIAD 5 — MARCH 3, 2009

Answers: [5A] **12** [5B] **101** [5C] **3** [5D] **1** [5E] **60**

74% correct

5A **METHOD 1:** *Strategy: Use a diagram to show what happens..*

The upper box shows Sara's age divided by 3 and then increased by 8. This restores her age, so 8 is $\frac{2}{3}$ of her age. Then $\frac{1}{3}$ of her age is 4, and **Sara is 12 years old.**



METHOD 2: *Strategy: Compare her age and 8 more than one-third of it.*

Sara's age is divisible by 3, so it is one of 3, 6, 9, 12, 15, and so on. Since this is 8 more than $\frac{1}{3}$ of her age, she is at least 9 years old. Build a table to compare her age with 8 more than $\frac{1}{3}$ of it.

Sara's age, possibly	9	**12**	15	18	21	…
1/3 of Sara's age	3	**4**	5	6	7	…
8 more than 1/3 of Sara's age	11	**12**	13	14	15	…

In all but one case, dividing her age by 3 and then adding 8 produces a different number. Sara is 12 years old.

FOLLOW-UP: *Three times my age is equal to 2 years more than my father's age. In 11 years, my age will be half of his age. How old am I?* [13]

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59%

5B **METHOD 1:** *Strategy: Use the fact that the common difference is 3.*
The increase of 3 in each successive term suggests comparing the numbers to the multiples of 3.

The given list is 5, 8, 11, 14, … and the multiples of 3 are 3, 6, 9, 12, … Each number in the list is 2 more than a multiple of 3. The greatest multiple of 3 that is less than 100 is 99 and 99 + 2 = 101. **The first number in the list that is greater than 100 is 101.**

METHOD 2: *Strategy: Determine some number in the list that is near 100.*
Begin with 14, the last number given. Add some convenient multiple of 3, say 30, to jump to other numbers in the list : 14, …, 44, …, 74, …, 104. Count backwards by 3 from 104 to get other members of the list: 101, and then 98. The first number in the list that is greater than 100 is 101.

22%

5C *Strategy: Use the algorithms for addition and multiplication.*
Since 0 is never a leading digit, O does not represent 0. In the addition, O + O does not produce a "carry", so O = 1, 2, 3, or 4. In the multiplication, 4 × O + a possible "carry" of 0, 1, 2, or 3 gives the two-digit number ending in FO.

Suppose O = 1. Then 4 × 1 + the carry = F1. No carry works.
Suppose O = 2. Then 4 × 2 + the carry = F2. No carry works.
Suppose O = 3. Then 4 × 3 + the carry = F3. This works if the carry is 1.
Suppose O = 4. Then 4 × 4 + the carry = F4. No carry works.

```
  O N E
+ F O U R
  F I V E

  O N E
×     4
F O U R
```

The letter O represents the digit 3.

Follow-Up: *Complete the solution of the cryptarithm. What is the four-digit number FIVE?* [1725]

16%

5D *Strategy: Determine the possible pairs of numbers on each tile.*
As shown in figure 1, list the 25 possible pairings, with the first letter representing the front of the tile. Eliminate the circled tiles which have the same letter on the front and back. This leaves two of each pairing. Cross out the second listing of each pairing. Figure 2 shows the ten remaining pairings.

figure 1

(AA)	AB	AC	AD	AE
~~BA~~	(BB)	BC	BD	BE
~~CA~~	~~CB~~	(CC)	CD	CE
~~DA~~	~~DB~~	~~DC~~	(DD)	DE
~~EA~~	~~EB~~	~~EC~~	~~ED~~	(EE)

figure 2

AB	AC	AD	AE
	BC	BD	BE
		CD	CE
			DE

Now consider the four tiles that have E on the back. Each row must have exactly one such tile, or else two tiles would be identical. The DE tile, however, is the *only* tile in the bottom row. You can point to the tile that shows the D and *know* the back has an E. **The fewest tiles you can point to is 1.**

Follow-Ups: *(1) Suppose the letter on the back does not have to be different from the letter on the front. Would the complete set still consist of 10 tiles?* [No. 15 are needed] *(2) Would you still be able to point to one tile and know the back contains an E?* [Yes. The E on the front would have to have an E on the back.] *(3) Suppose there are more than 5 letters. Would you still be able to find the F (or G or ...) on the back in one try?* [Yes.Use the above reasoning.]

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5E **METHOD 1:** *Strategy: Evaluate each expression.*

$8 \diamondsuit 5 = \underline{(8 \times 8) - (5 \times 5)} = 64 - 25 = 39$

$5 \diamondsuit 2 = \underline{(5 \times 5) - (2 \times 2)} = 25 - 4 = 21.$

The sum is $39 + 21 =$ **60.**

METHOD 2: *Strategy: Eliminate unnecessary terms.*

$(8 \diamondsuit 5) + (5 \diamondsuit 2) = \underline{(8 \times 8) - (5 \times 5)} + \underline{(5 \times 5) - (2 \times 2)}.$

Notice that you are both subtracting and adding (5×5).

$(8 \diamondsuit 5) + (5 \diamondsuit 2) = \underline{(8 \times 8) - (2 \times 2)} = 64 - 4 = 60.$

***FOLLOW-UPS**: (1) Suppose $(27 \diamondsuit 23) + (23 \diamondsuit 12) = (27 \diamondsuit N)$. What is the value of N?* [12]
(2) What is the sum of $(100 \diamondsuit 97) + (97 \diamondsuit 94) + (94 \diamondsuit 91) + \ldots + (4 \diamondsuit 1)$? [9,999]

***NOTE:** Other FOLLOW-UP problems related to some of the above can be found in our books "Math Olympiad Contest Problems", both Volume 1 and Volume 2, and "Creative Problem Solving in School Mathematics."*

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