3A *Time: 4 minutes*

How many square units bigger is a 5 × 5 square than a 4 × 4 square?

3B *Time: 4 minutes*

A bicyclist takes 2 minutes, 30 seconds to travel a distance of 1 mile. At this rate, how many miles will the bicyclist travel in 1 hour?

3C *Time: 5 minutes*

In a parking lot are 60 cars, all of which are black or blue and all of which are Fords or Dodges. There are 25 black Dodges. There are 18 blue cars. There are 25 Fords. How many blue Fords are there?

3D *Time: 7 minutes*

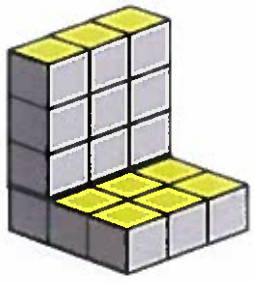
A, *B*, *C*, and *D* represent the numbers 2, 4, 6, and 8, in some order.

We also know that $\frac{A+B}{C+D}=\frac{D+C}{B+A}$.

Find the value $(A \times B) + (C \times D)$.

3E *Time: 7 minutes*

A Super-Brick is formed by arranging nine 1cm × 1cm × 1cm cubes into a 3cm × 3cm layer, 1cm thick. Two Super-Bricks are glued together into the L-shape shown. All faces of the L-shape are completely painted. What is the area of the painted surface in square centimeters?



Please fold over on line. Write answers on back.

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Division E — Mathematical Olympiads for Elementary & Middle Schools — January 12, 2016 — Contest 3

SOLUTIONS AND ANSWERS

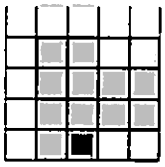
3A

3A METHOD 1: *Strategy: Find the area of each square and find the difference.*

The area of the larger square is 5 × 5 = 25 square units.
The area of the smaller square is 4 × 4 = 16 square units.
The difference of the two areas is 25 – 16 = **9** square units.

METHOD 2: *Strategy: Draw the smaller square inside the larger square.*

Find the number of squares outside the smaller square but inside the larger square. There are 9 square units not shaded.



FOLLOW-UPS: (1) How many square units bigger is a 104 × 104 square than a 103 × 103 square? [207] *(2) Betty is tiling an 8-foot by 8-foot floor with 12-inch square tiles. She buys some and finds that she needs 15 more tiles to completely cover the floor. How many tiles did she initially buy?* [49]

9

3B

24

3C

3B METHOD 1: *Strategy: Simplify the time it takes to bike 2 miles.*

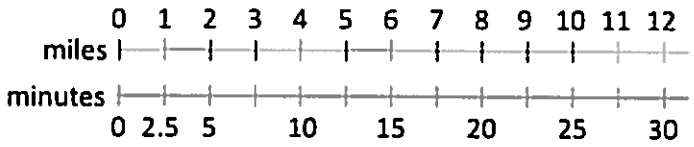
2 min 30 sec + 2 min 30 sec = 5 minutes
5 minutes → bike 2 miles
10 minutes → bike 4 miles
30 minutes → bike 12 miles
60 minutes → bike **24** miles

8

3D

METHOD 2: *Strategy: Convert all the times to minutes and use a number line.*

The cyclist traveled 12 miles in 30 minutes or 24 miles in 60 minutes which is 1 hour.



40

3E

METHOD 3: *Strategy: Simplify the time it takes to bike 2 miles.*

There are 120 seconds in 2 minutes, so the bicyclist takes (120 + 30 = 150) 150 seconds to travel 1 mile. There are 3600 seconds in 1 hour (60 sec/min × 60 min = 3600 sec).

Let x = the number of miles traveled in 3600 seconds so the proportion is $\frac{x}{3600} = \frac{1}{150}$.

Multiply both sides of the equation by 3600, hence, x = 3600/150 = 24 miles in 1 hour.

FOLLOW-UP: Cary travels 2 miles in 20 minutes. Cindy leaves 5 minutes later from the same place and following the same route at a rate of 2 miles in 15 minutes. How long will it take Cindy to catch up to Cary? [15 minutes]

54 sq cm

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3C **METHOD 1:** *Strategy: Use the process of elimination.*

If there are 60 cars and 18 are blue, 60 – 18 = 42 are black. Twenty-five of the 42 are Dodges so 42 – 25 = 17 are black Fords. If 17 of the 25 Fords are black, then 25 – 17 = 8 must be blue. There are **8** blue Fords.

METHOD 2: *Strategy: Complete a table.*

Create a table of the four possible cars:

The bold numbers in the table are the given values.

The italicized numbers were found in the following order: 35, 10, 8.

	Black	Blue	Total
Dodge	**25**	*10*	*35*
Ford		*8*	**25**
Total		**18**	**60**

FOLLOW-UP: A class of 24 students is treated to ice cream cones. The only flavors available are vanilla and chocolate and the only cones available are sugar and waffle. If 10 students choose chocolate in a waffle cone, 7 students choose vanilla and 10 choose sugar cones, how many students choose vanilla in a sugar cone? [3]

3D *Strategy: Apply the commutative property of addition.*

Recognize that $A + B = B + A$ and $C + D = D + C$. Therefore the two equal fractions are reciprocals of each other. The only number that equals its reciprocal is 1. Thus $A + B = C + D$. The only arrangement of the four given numbers is 2 + 8 = 4 + 6 = 10 and therefore 2 × 8 = 16 and 4 × 6 = 24 so $(A \times B) + (C \times D) =$ **40**.

FOLLOW-UP: Let A, B, C, and D be single digit odd numbers. If A + B + C + D = 20, find the value of A × B × C × D. [189]

3E *Strategy: Find the surface area of a Super-Brick and subtract the areas of the glued sides.*

The surface area of each Super-Brick is the sum of the areas of the 6 faces.

Area of the front = area of the back = 3 × 3 = 9 sq cm

Area of the top = area of the bottom = 1 × 3 = 3 sq cm

Area of left side = area of right side = 1 × 3 = 3 sq cm

The sum of all 6 faces = 2 × (9 + 3 + 3) = 30 sq cm. Since there are 2 bricks the total surface area is 60 sq cm. One 1 × 3 face is hidden on each brick, so the visible area is 60 – (3 + 3) = **54** sq cm.

FOLLOW-UP: Find the total surface area of a 3 × 3 × 3 cube if the middle column of 1 × 1 × 3 is removed. [64]

NOTE: Other FOLLOW-UP problems related to some of the above can be found in our three contest problem books and in "Creative Problem Solving in School Mathematics." Visit www.moems.org for details and to order.